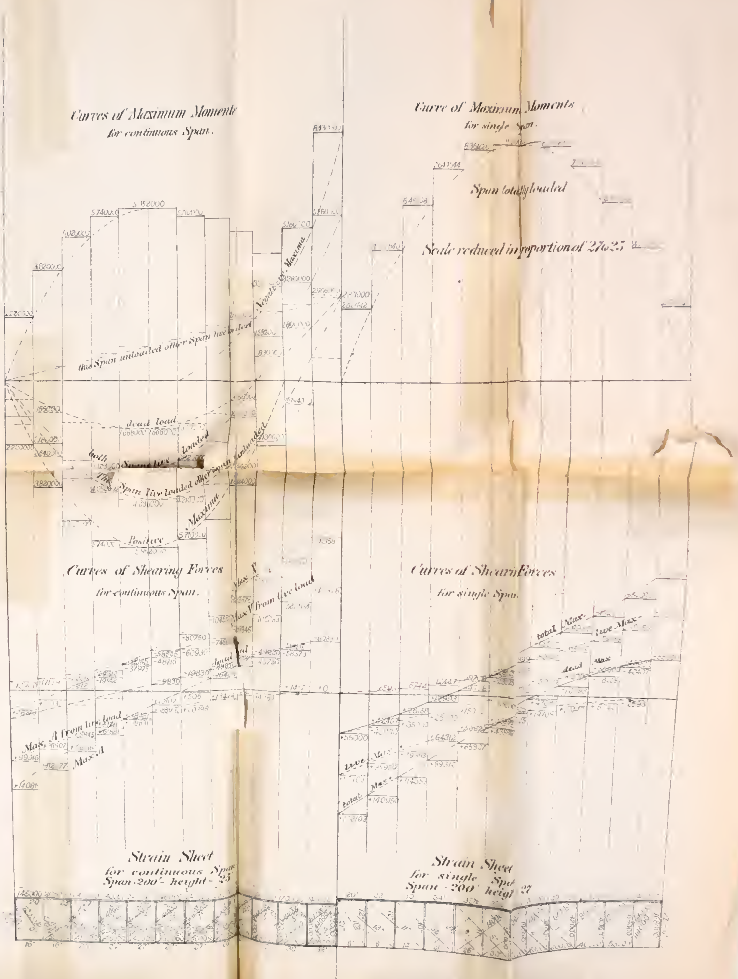
Curves of Maximum Moments
for continuous Span.
Curve of Maximum Moments
for single Span.
Span totally loaded
Scale reduced in proportion of 27 to 25
this Span unloaded other Span live loaded
Negative Maxima
dead load
both Spans live loaded
This Span live loaded other Span unloaded
Maxima
Positive
Curves of Shearing Forces
for continuous Span.
Curves of Shearing Forces
for single Span
Max V from live load
dead load
Max A from live load
Max A
total Max
live Max
dead Max
live Max
total Max
Strain Sheet
for continuous Span
Span 200' height 25'
Strain Sheet
for single Span
Span 200' height 27'

PRACTICAL TREATISE

ON THE

PROPERTIES OF

CONTINUOUS BRIDGES.

By CHARLES BENDER, C. E.

MEMBER OF THE AMERICAN SOC. CIVIL ENGINEERS.

NEW YORK:

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1876

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The change in the strains of continuous bridges from tension into pressure must be provided for by proportioning the members to the sum of both strains. Different systems of bridges must be compared by giving to each its proper and most advantageous proportions, but not by supposing all to be of the same depth.

Continuous plate girders not only are really more economical than single span plate gilders, but the theory also is more in conformity with the latter mode of construction. Many French engineers, in accordance with the supposition of theory, applied uniform cross-sections of flanges.

Economy of such girders, with constant flanges, by lowering of middle supports, or in case of three and more spans by suitable proportions of spans.

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PREFACE.

A paper by the author, being a critical examination into the merits of continuous girders, was prepared six years ago, but its presentation to the American Society of Civil Engineers was delayed till the last spring. In this paper the subject for the first time is found to be based without the use of higher calculus on one simple geometrical relation, forming the connecting link between single spans and continuous bridges.

The same paper, increased with further data resulting from its discussion, is compiled into the short treatise now presented.

It is hoped that it may contribute to clear opinions as to the real merits of the systems of single and continuous spans, and would lead to a more thorough understanding of the nature of each.

The subject having never before been treated in this light, it is believed that railroad engineers will not unfavorably receive the results of special studies which have occupied

a period of many years, and which in the main are:

That in addition to the sensitiveness of continuous bridges, the economy claimed for them does not exist either theoretically or practically in all instances in which the construction of properly designed compound single span trusses is not limited as to their depths. As a result of these conclusions, there would seem to be one more good reason that the most valuable time of polytechnic students should not be unnecessarily wasted by entering deeply into a theory which more essentially is of mathematical and historical interest.

C. BENDER, C. E.
Member American Society of Civil Engineers.

New York, October 12, 1876.

APPLICATION OF

The Theory of Continuous Girders

TO ECONOMY IN BRIDGE BUILDING.

Lately the introduction in this country of continuous girders has been suggested on the plea of greater economy than, it is asserted, can be obtained under application of the highly perfected and simple American trusses. The majority of advocates of that system who are now devoting their time to theory, though well acquainted with the mathematical part of the subject, have omitted some practical bearings which would be necessary to enforce their assertions. In reality, mathematical investigations of the subject of continuous girders require not a very high degree of training in analysis. It needs but the execution of the integration of one single equation, which execution may become lengthy and tedious, and may require much patience. But once the one mathematical idea

of that equation be understood, the rest of the work is common and mechanical algebraic labor.

If it were true that continuous girders give more economy than the system in use in the United States, it would certainly be a heavy charge against the engineers who have made the art of bridge building their specialty, and who study their profession with all earnestness. Many will deny, at the outset, that this charge is just ; for the sake of others, the author proposes to show that the American practice of bridge building hitherto, has been in the proper direction towards other improvements, and that the theorists who wish bridge builders to follow their advice have studied the subject in but one of its bearings, and have omitted to examine closely their premises as well as their conclusions.

The author believes it is not only desirable, but necessary that this question should be fully discussed, from various reasons. Practical engineers generally do not place much confidence in long formulæ, and if they once have studied mathematics thoroughly they lose the taste for these studies after some time of practice, since they have convinced them-

selves as to the futility of ultra refined theoretical speculations. These engineers will not be very likely to adopt structures whose calculation of strains would waste so much valuable time. But these engineers could not prevent a new method of construction in time becoming fashionable, whether correct or not, as long as it were founded on some elegant theory and seemingly led to economy. For, under our large factors of safety, we can commit many sins in construction before they are found out. Again, there is always a number of men who, because they do not understand abstruse calculations and formulæ, rather than admit this fact, publicly endorse them warmly. And finally, when in polytechnic schools for a number of years, a certain theory has been thoroughly studied with zealous assiduity, a little army of its admirers will fill positions in railroad and in public engineering offices, anxiously waiting for the first opportunity towards introducing into practice what they consider the finest jewel of their technical knowledge. The author frankly admits once to have been of this number. But after studying the subject of continuity of trusses for several years, and a

careful examination of its suppositions he found himself compelled to admit that the theory is not correct scientifically, and does not agree with the physical laws of elasticity of iron.*

We are now prepared to prove, that for medium spans, say of 200 feet, the construction on the principle of continuity leads to *greater* truss weights in addition to greater cost of workmanship than are required by the use of single spans with improved details.

This last result is very important indeed,

* Six years ago, in a paper written for the German Society of Engineers, *Verein Deutscher ingenieure* in Berlin, which was translated into English and published two years ago in the *Railroad Gazette* of New York, the author stated:

"The writer of these lines himself had for some time thought that it might be possible, by application of pin joints, by reducing the number of parts, by the use of proper scales and adjustments for the regulation of the pressures on the three or more piers of a continuous bridge, and by the use of scientifically correct and complete formulæ, to produce reliable continuous trusses, by means of which the large rivers of this country could bespanned without the use of false works."

"With a great deal of labor he had constructed an analytical expression, which embraced the relation of the moments of flexure over three consecutive piers of a continuous girder. In this formula, due attention was given not only to the deflections caused by the chords, but also to those due to the tensile and compressive members of the web system; also the actual section of each separate member was introduced. It therefore did away with two errors of the formulæ generally quoted in books, which are only applicable when the girders are very shallow and when the web is a

for if it were possible, under application of the principle of continuity to arrive at an economy in weight and cost, there would be a large market for this article however objectionable the mode of construction; for, railroad officers in the majority of instances will be led by the consideration of first cost; especially since, in bridge building (thanks to our factor of safety) many errors remain unpunished for a long time, continuous girders with their delusive theory and deceptive stiffness under application of lattice and rivets would gain a wide market.

plate, and which even under those suppositions do *not* coincide very satisfactorily with experiments."

"Notwithstanding the theoretical improvements mentioned, it was finally found that the labor spent in finding said formula had been in vain, from a reason which in Europe, as far as known, has not received any consideration. It is the great variability of the modulus of elasticity, which in the formulæ of the books is supposed to be a constant value of about 25,000,000 pounds per square inch."

"But the writer has tested, during his presence at the Phœnix Iron Works, many thousands of eye-bars, made for actual use in bridges, and found that the modulus of these members is very changeable, namely from 18,000,000 to over 40,000,000 pounds per square inch, so that small sections give the lowest and large sections the greatest figures. The same result was obtained by the Canadian engineer who inspected the iron for the International bridge near Buffalo, as well as by Mr. B. Nicholson, who was sent to Phœnixville by the government officers of the United States to inspect the iron for the Mississippi bridge at Rock Island."

The theory of continuous girders, as given in text-books, does not always permit the philosophy of the principle involved to be clearly seen: its representation generally is rather obscure. In order to explain this principle as clearly as possible, the author worked out a new method of treating the subject. The results under this treatment naturally must agree with those derived from the application of the general theory of the elastic line which, in the last century (1744), was first given by Leonard Euler of Basel, then member of the Academy of Science in Berlin, which, by Navier, early in this century, was propagated among engineers, and lately was somewhat simplified by Henry Bertot in France.*

* Jacob Bernouilli having in the year 1695 given the notion of the "*neutral line*," tried in 1705, shortly before his death, to find the equations and properties of the "elastic line." In this he did not succeed, but Leonard Euler in his book "de curvis elasticis" (Lausanne and Geneva), 1744, solved this problem, showing that for flat elastic curves the second differential co-efficient is proportional to the moment of flexure of exterior forces. P. S. Girard, in his work "Traite analytique de la resistance des solides, 1798," page 50, &c., translated Euler's treatise from the Latin into the French language, and he adds as an application of Euler's theory the investigation of a beam fixed at both ends. Eytelwein and Navier extended this labor to the beam continuous over three and more supports. Henry Bertot in France, in the year 1855 (Comptes Rendus de la societe des Inge-

I.—The general Principle involved in the Theory of Continuous Girders.—We first consider a number of single spans of the lengths l_1, l_2, l_3, l_4, touching each other respectively over the piers B, C, D, E. We suppose each span to be loaded in any conceivable or desired manner ; in consequence, each span would deflect so as to form certain curves as indicated by dotted lines. The lower chord would not remain straight, the end-posts would not remain vertical. Differing with the nature of the material with the sectional areas of the members of the bridge, with the loads imposed upon them, the trusses would show certain angles γ_1, δ_1, γ_2, δ_2, γ_3,

nieurs Civils de Paris, page 278, &c.), for the first time gave what is called "the theorem of the three moments," which later (1857), and independently of Bertot, was found by Clapeyron and Bresse in France, and by the English engineer, Heppel, in the year 1858, in India.

In the United States Colonel Long, the well known inventor, in the book on his patent (1838) truss, edited in 1841 in Philadelphia, probably for the first time has applied the principle of continuity to wooden skeleton trusses and numbers of such wooden bridges since then were built in this country.

Mr. Pole in 1852 applied the theory to a bridge over the Trent, consisting of two continuous spans of 130 feet each, and this engineer also did the analytical part of the calculation of the strains of the Britannia bridge, such as contained in Mr. Edwin Clark's famous work. (Vide Transactions I. C. Engineers, Vol. XXIX.)

δ_3, &c., &c., of the end-posts with their originally vertical positions. Now, suppose you draw together the top points of the end-posts over the piers B, C, D, E, and press apart the bottom joints of these posts, so that not only the top but also the bottom chords of the adjacent spans would touch each other; or, in other words, insert certain tensile forces into the upper chord, and equally large compressive forces into the lower chord of each truss. Thus each truss by a certain unknown moment of flexure would artificially be bent upward in such a manner that certain angles α_1, β_1, α_2, β_2, α_3, β_3, would be produced, were the dead and the live loads of the trusses removed.

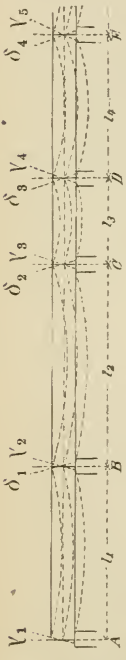


Fig. 1.

When at each central pier, the desired continuity, consisting of connection of the top and bottom chord ends, separated under the dead and live loads alone, but overlapping each other un-

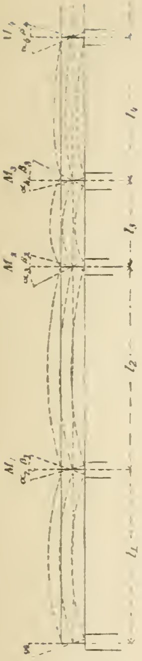


Fig. 2.

der the action of the moments M_1, M_2, M_3, &c., is effected, this law, must obtain, namely: *the sum of the angles of deflection at any central pier caused by the (dead and live) loads on two adjacent trusses must be equal to the sum of the angles of elevation, caused by the unknown moments artificially applied at the three piers of the contemplated spans.* This is expressed algebraically:

$$\left.\begin{array}{l}\delta_1+\gamma_2=\alpha_2+\beta_1\ ;\ \delta_2+\\ \quad\gamma_3=\alpha_3+\beta_2\\ \delta_3+\gamma_4=\alpha_1+\beta_3,\ \&c.\end{array}\right\}(I.)$$

In these equations the left sides are functions of the dead and live loads, and the right of the unknown moments, M_1, M_2, M_3, &c.

For each intermediate pier there is one equation and one unknown moment. The number of equations equals the number of unknown moments, which equals the number

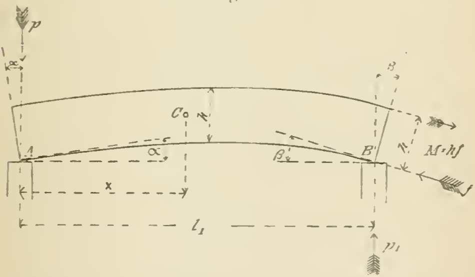
of spans of the continuous bridge, less one. For n spans there are $(n-1)$ equations and $(n-1)$ unknown moments. These $(n-1)$ equations are therefore sufficient to solve the problem which is identical with finding the unknown moments. From the theory of single spans, which again is founded only on the law of the lever, we calculate angles of deflection like δ, γ, α, β, and therefore the above law, expressed by the very plain equations (I), indicates how to derive the strains of continuous girders from those of single spans. This law, as it were, is the tune for all the rest of variations relating to continuous girders.

If we express the angles α and β, by the unknown moment acting on two continuous spans we arrive directly at the formula of Henry Bertot, improperly ascribed to Clapeyron, which is found after a tedious process of integration. *Bertot's formula in fact expresses only the geometrical law that the sum of the angles of deflection must be equal to the sum of elevations due to the moments* M_1, M_2, M_3, *&c., over the central piers.*

Though we suppose the reader to be acquainted with the theory of single span

bridges, in the sequel we shall develop a few rules belonging to this theory sufficient for us to directly write down the formula from which we calculate the moments, M_1, M_2, &c., and consequently the strains. Before doing this we first wish to more fully explain the next consequence of the principle of continuous girders. A span $A\ B$: without weight is resting on two supports A and B; at B a moment M_1 equal to forces f,—f, with the lever h acts on the chords. This moment is counteracted by a force (weight) p, holding the truss-end A to the pier. Though

Fig. 3.



the force p, holds down the end A, yet the moment $M_1 = f\,h$ will cause a convex elastic curve, so that the end posts which originally were vertical, are made to form

angles α and β, to their vertical positions.

Since a moment of flexure can only be neutralized by another opposite moment, there must exist another force,—p_1, acting on the pier B, which in combination with $+p_1$, on the lever l, equals precisely $M_1 = f\,h$; in other words, p_1 must be equal to $\frac{M}{l_1}$ and $M_1 = p_1 l_1$.

For any section C, of the beam $A\ B$, the moment of flexure is equal to the force p_1, multiplied by the distance x, and the greater x is, the greater the moment of flexure in C. When x becomes equal to l_1, the maximum of the moment is reached, namely $M_1 = p_1\ l_1$, whilst at A, the moment of flexure acting on the beam is zero, because x is zero. And as the curvature of a beam increases directly as its moment, the beam is not bent at all at A, but is gradually bent more and more the nearer we come to B.

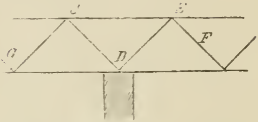
From what has been said, this law can be deduced: that the application of a moment M_1, to the central pier of an end truss span of a continuous girder does call forth two forces $+p_1$, $-p_1$, which, however, do *not* alter the *sum* of the reactions A and B of this span in whatever manner it may be loaded.

The moment M_1 reduces the pressure on the pier A, but only by increasing with the same amount—p_1, the pressure on the pier B. From this observation it further follows that by the principle of continuity, no load resting on an end span can be carried over to the next span, but that the sum of these loads always is neutralized by the two nearest piers between which it acts. The distribution only of the reactions, which for single spans is governed by the law of the lever, in the end spans of continuous girders is modified.

What has been said of a single span acted upon by one moment M_1, is equally true, if on its other end another moment M_2, would act. All we need do is to add together the effects due to each separate moment. Therefore, also, any load acting on a middle span of a continuous girder is taken up by the two nearest piers. Also, in this instance, the sum of the two partial reactions belonging to this span on these piers, equals the total load between them. Only the proportion between these reactions, by the principle of continuity is modified.

This result could have been anticipated from the following consideration: The strut

$C\ D$, Fig. 4, carries down to the pier D the



loads due to the span G. As long as the bearing D is inelastic, the diagonal $D\ E$ can not be drawn down and the vertical pressures carried by the members $C\ D$ and $E\ D$ must be directly annihilated in D. The case would be very much different if the pier D were elastic,* for then there would arise a deflection of the truss $G\ F$ in D, and a portion of the shearing force from one truss could travel to the next one.

Having learned that the moments M_1, M_2, M_3, cause the existence of pairs of forces $+p_1 - p_1, +p_2, -p_2, +p_3 - p_3, +p_4 - p_4$, &c., it is very easy now to express the exact value of the moments by the forces p_1, p_2,

* The supposition of elastic supports, consisting of systems of springs, was investigated by the writer in 1867, in an extensive series of calculations, with a view to determine whether thereby any economy could be secured, and with the intention to use these springs as adjusting and weighing apparatus of the elastic reactions. The result was, that the variable positions of the movable load so much reduced any gain in the chords that the additional expense of the systems of springs left no economy for a structure of this kind.

&c., and their levers, l_1, l_2, l_3, &c. These values simply are (n being the number of spans):

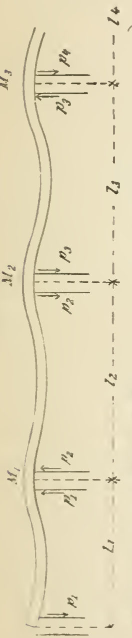


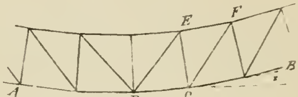
Fig. 5.

$$\left.\begin{aligned} M_1 &= p_1 l_1. \quad M_2 = M_1 - p_2 l_2 = p_1 l_1 - p_2 l_2. \\ M_3 &= M_2 + p_3 l_3 = p_1 l_1 - p_2 l_2 + p_3 l_3. \\ M_4 &= M_3 - p_4 l_4 = p_1 l_1 - p_2 l_2 + p_3 l_3 - p_4 l_4, \text{ \&c., \&c.} \\ M_{n-1} &= M_{n-2} + p_{n-1} l_{n-1} = p_1 l_1 - p_2 l_2 + \ldots + p_{n-1} l_{n-1} \end{aligned}\right\} (II.)$$

and, finally, $O = M_n =$ sum of moments $(pl) = \Sigma(pl)$, (from p_1 to p_n). The last moment M_{n-1} is equal to $\Sigma_{n-1}\ (pl)$, and also is equal to the moment $p_n\ l_n$, so that the $(n-1)$ moments are sufficient to calculate the n forces $p_1\ p_2\ .\ .\ .\ p_n$. After these preparations we proceed to the values of the angles $\alpha_1\ \beta_1\ \gamma_1\ \delta_1$; $\alpha_2\ \beta_2\ \gamma_2\ \delta_2$; &c., &c. Can these values really be calculated? With the answer of this question in the affirmative or negative stands or falls the

whole theory of continuity of girders: and this question is followed by another: can we calculate with a sufficient degree of reliability the elastic line of a single span bridge? The originally straight truss A B through

Fig. 6.



the influence of live or dead loads has received a deflection. The angle A C B, which originally equalled 180°, has been altered, and the alteration x C B was caused by the loads.

The question then arises, can the angle xCB be calculated with a sufficient degree of reliability? The alteration of the angles around C is equal to the sum of the alterations of the separate angles. Each angle is altered because each side of each triangle has been altered in length. Some sides have been shortened under pressure, some have extended under tension. These extensions and compressions are very small, and it is very difficult to measure them.

If we knew the alterations of the sides,

also hence the altered angles, calculation would be possible. But to determine the extension or compression of each side of each triangle, it is necessary to know for each side, the total strain and the exact value of the cross-section, and precisely how much each side (each member) will extend or compress under the action of a ton per square inch. The extension or compression of a member of a girder therefore depends on the strain per square inch at any point of this member, and on quality of the material.

For plain, single span bridges with hinged joints, the law of the lever teaches us to calculate the total strain in any member. We have reduced, in principle, the problem of continuous girders to a combination of problems on single spans. Hence also for continuous girders we could determine those total strains, if it were possible to calculate the deflections of single span girders. But we do not know beforehand the sectional areas of the different members of a continuous girder; on the contrary, it is just our problem to find those sections which are most suitable.

The theory of continuous girders as treated in text-books, leaps over this difficulty by

making an arbitrary supposition, namely, that all sections are equal. We shall have occasion to show that this is theoretically incorrect, and causes an error of as high as 15 per cent. of the calculated values, and this error probably is as large as the theoretical gain claimed for the chords of continuous girders.

As far as the nature of the material is concerned, we know that extensions and compressions are proportional to the strains per square unit, and that, in order to find them, the strain per square unit must be divided by a coefficient proper to the material which is called the modulus, and the quotient thus obtained must be multiplied by the original length of the member. The next question is, therefore, do we know the value of this modulus for the material used?

This question must be answered by experiment. It belongs to the science of natural philosophy. The physical suppositions upon which mathematical investigations are based should always be founded on undeniable facts, since the truth of these suppositions is the *conditio sine qua non* of the value of the resulting formulæ. For the supposition being reduced to a mere hypothesis, it is wholly

indifferent by what brilliant and elegant analytical or graphical method the deficiencies of the foundations are hidden.

The theory assumes that the modulus of elasticity for any part of a girder is a constant value unaltered to any noticeable extent by manufacture; or by rivets, covering plates, different sections of rolled iron, or by thickness of the metal, &c., &c. Practical men will be likely to demand that the truth of this broad hypothesis be demonstrated. Therefore, previous to calculating, let us examine the hypothesis of the theory of continuity. Nowhere, in applied science, the necessity for such examination is more urgent.

The extensions and compressions allowed in practice are very small quantities. Defective testing machines, unacquaintance or unfitness of experimenters to such delicate work, temperature, variability of manufacture of material under test, variability of the chemical composition, density, uniformity, &c., are causes of great errors. Often the number of tests were too small to draw therefrom any justified conclusion, or the elements of time and motion have been neglected, or experimenters overlooked other important

elements altogether. There are also instances that experimenters were not impartial, and that theories have been formed first, and experiments have been arranged afterward to suit. Experimenters may reject results which in their opinion seem untrustworthy, while they report and elaborate others which to them seem probable because favorable to their theory. Or experiments were made on one sort of material and applied to material of quite a different nature and section. It is by no means an easy labor to conduct trustworthy experiments on the elasticity of material, and Professor Wullner* is perfectly correct in saying: "The examination of the elasticity of solid bodies is one of the most difficult in the whole science of natural philosophy. In order to conceive its laws sufficiently, the most intricate mathematical investigations and the most subtle experiments are required." These two conditions rarely are found combined.

We shall soon see that the theory of continuous girders was built up exclusively by men of purely mathematical capacities, and

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* In the first volume of his work on physics.

that they did not begin with that cautious examination of their suppositions which is demanded by true science as much as by practice. It is true that some theories have been worked out before the basis properly was investigated, and are applicable because the results of such theories were *finally* tested. But we have no such experiments on continuous trusses. For the measured deflections of executed continuous bridges are neither sufficiently exact nor are they of use for our purpose, since we do not know what permanent sets were produced after removal of the false-work, nor was the modulus of each member previously examined and recorded.

This remark also applies to single spans, and the question therefore again most pertinently arises, whether the exactly calculated deflection of a single span ever did agree with its real deflection. Logic compels us to acknowledge that such coincidence is impossible, save by sheer accident ; for, who has examined the modulus of each finished riveted compression member, or of each rod or bar of a bridge ? Who, therefore, was capable of calculating the deflections ? Moreover,

these calculations generally have been faulty,* and if engineers assert that the deflections of their bridges agree with their calculations, either the first were erroneously calculated or the second not well observed.

Since we have no experiments on the qualities of finished continuous girders, the more should we examine the results of experiments on the values of moduli of iron and steel. These experiments, fortunately, are very numerous indeed, and gives us all information wanted, whilst, unfortunately, theorists on continuity have not considered it worth while to consult this great quantity of scientific material previous to entering into mathematical speculations.

II. Experiments on the Values of the Moduli of Iron and Steel.—Bornet, in France, over 40 years ago, made experiments on rods of chain iron 0.2 inches in diameter

* The influence of the webs on deflections generally is neglected. With the exception of by Schwedler (see official Engineering Periodical—*Zeitshcrift fur Bauwesen, Berlin*), no successful attempt was made to examine the influence of web posts and diagonals. In the sequel we shall show that this influence is enormous and very perplexing; in fact, that calculations of continuity of bridges, without properly considering the webs, are worse than worthless. The calculations there shown were first developed by the writer in 1869, and are not found elsewhere.

and 21 feet long. The original modulus found was 35,500,000 pounds, and under strains of 20,000 per square inch, it decreased to 28,500,000 pounds. Ardant, in France, for soft annealed wire, up to strains of 35,000 pounds per square inch, found the modulus to be 24,000,000 pounds, and for hand-drawn wire, up to strains of 42,000 pounds per square inch, it was 27,300,000 pounds. Whilst of wire, Ardant did not perceive any lowering of the modulus up to strains of 42,000 pounds, Bornet remarked a diminution beginning with strains of 8,500 pounds per square inch. It is not stated by Morin, who reports these results, in what manner the experiments were made.

Hodgkinson made (only) two tensile experiments on long rods to determine their extensions with exactness, and found the orginal modulus of one bar equal to 23,900,000, and of the other to 22,400,000 pounds ; Edwin Clark gives 29,000,000 pounds. Vicat's experiments on hard wire with 0.1 inches diameter, resulted in a modulus of 28,200,000, and for wire well annealed, 20,660,000 pounds. Morin for hardened wire gives 28,100,000, and for annealed wire 22,400,000 pounds.

Experiments of a more practical value were made by Malberg, in Prussia, on occasion of his building the Mulhelm suspension bridge. The bars for this structure were made by Herr Daelen. The iron was of best German stock, the puddle loops well hammered, rolled, piled, and re-rolled. All bars were of the same stock, same make, same length, same sectional shape and area. Their moduli, however, varied from 20,000,000 to 27,000,000 pounds, which gives a difference of 35 per cent. for the same kind and section of bars.* This great variability of moduli of bars of even the same shape and material, was further noticed on occasion of the construction of the Vienna Railroad suspension bridge, where bars of the same modulus were put into the same panels.

The author had occasion to test many thousand of eyebars, up to about forty feet length, and varying in section from 1 to 14.25 inches square. The moduli of these bars varied much according to their cross-sections, and were from 18,000,000 to 40,000,000

* Herr Daelen is an authority, known by his universal mill, a treatise on the art of shape rolling, and his invention of weldless rolled eyebars, known as Howard's Patent.

pounds, and even higher. These results were confirmed by other inspectors of bridge work, for instance, by Mr. B. Nicholson.*

We turn to moduli of steel. Morin, for steel 0.167 inches square, from Petin & Gaudet, found 31,000,000 and 31,800,000 pounds. Direct tensile experiments on Krupp's steel, by Woehler, gave on the average, a modulus of 32,560,000 pounds, whilst from flexure he calculates 31,100,000 pounds.

Prof. Staudinger, of Munich, has made careful tests on Bessemer metal,† when the moduli were found to be independent of the quantity of carbon combined with the iron.‡ The following table contains the results of his experiments :

* At Phœnixville, Pa.

† From the Pernitz Works in Austria.

‡ The quantity of carbon rose from 0.14 to 0.96 of one per cent. Metal with 0.14 is soft iron, with 0.19 to 0.30 it is granular iron (of a fine grain) or hard iron, then comes soft steel, which increases in hardness with the carbon contained.

Moduli in Million Pounds per Square Inch.

Carbon—per cent.	0.14	0.19	0.46	0.51	0.54	0.55	0.57	0.66	0.78	0.80	0.87	0.96
Tensile moduli, short pieces test	32	30.4	32	31.4	30.6	31.5	31	32.4	32.5	30.5	31	31.2
Compressive moduli	38.2	37	32.7	32.4	36	33.4	32	35.6	32.4	32.3	31.5	32.7
Tensile moduli of bars	32.2	.....	.....	.....	33.4	.....	32	32.7	.....	.....	.....	.....
Do. screw rods 13 ft. long	28.8	.....	.....	.....	29.5	.....	28.9	31.5	.....	.....	.....	.....
Moduli by flexure	28.4	29.1	28.4	30	28.8	30.3	29.3	32.1	30.4	33	30.7	29.3
Tortional moduli	.....	12.15	12.1	.....	12.1	.....	11.9	12.3	12.1	12.7	12.1	11.4

This shows differences of moduli as large as 33 per cent., for the same class of metal. It also proves that Hodgkinson is wrong in quoting the compressional modulus of wrought iron lower than the tensile modulus; it again gives evidence that the softest metal (that contains 0.14 per cent. of carbon) may give a higher compressional modulus than even the hardest steel of this table.

At the Vienna Exposition, a set of test-pieces could be seen,* which showed as follows :

Carbon—per cent.......	1.	0.75	0.5	0.28	0 12
Specific gravity.........	7.83	7.84	7.85	7.86	7 88
Tensile modulus (millions pounds........	25.1	24.6	27.7	24.9	26.1
Ultimate tensile strength—	90 000	80 000	70 000	67 000	65 000 pounds

The modulus of this class of steel and iron was, in the average, noticably lower than those for Ternitz iron and steel, the difference being about 20 per cent.

B. Baker made some experiments on steel bars previous to his experiments on crippling strength and found the modulus from 29,100, 000 to 37,330,000 pounds, which result shows a difference of 28 per cent. He says† "Every practical man who has noted the behaviour of iron girders under bending stresses, knows whilst one girder may deflect a certain amount under the test, another one precisely similar and placed apparently under precisely the same condition, may deflect some 30 per cent. more or less."

Hodgkinson made two direct experiments

* Bessemer metal from the Reschitza Works in Hungary.

† In his book on Beams and Columns.

the compression moduli of iron, and found 1[illegible],200,000 and 21,000,000 pounds. These two experiments strengthened Hodgkinson in his belief of the correctness of his theory as to a weakness of wrought iron under crushing stresses, whilst they only prove how easily an experimenter may be misled. Duleau, in France, directly measured the compressions of fibres in comparison with their extensions, which he, differing from Hodgkinson, found to be exactly equal.

Very valuable hints as to the qualities of iron can be derived from experiments on flexure, which can be conducted easily with sufficient accuracy. Morin, by such, determined the following moduli :

For iron from works near Rouen...... ...	31,800,000 pounds.
" " " Jackson Petin & Gaudet...	28,400,000 "
" " " Ale' Lik (Algeria)..........	28.960,000 "
" English crown bars.	23,440,000 "
" French I beams with equal flanges..	29.330,000 "
" " " " unequal " ..	24,400,000 "
" beams also from Dupont & Dreyfussin Ars sur Moselle...... ..	26,00,0000 "
" beams also from Dupont & Dreyfussin, equal flanges..............	23,600,000 "
" beams also from Dupont & Dreyfussin, unequal flanges....... ...	23,000,000 "
" beams also from Dupont & Dreyfussin, same beam reversed......	23.000,000 "

Here again we have differences of moduli

amounting to 39 per cent., and for the same class of iron (Lorraine beams) of 27 per cent.

Thomas D. Lovett* has lately furnished an elaborate series of experiments on compression members, such as actually used in the bridges of the Cincinnati Southern Ry. Up to the time of his report† 30 compression members had been tested and broken; their moduli varied from 19,300,000 to 34,600,000 pounds.‡

Experiments on hollow wrought iron tubes made by Hosking gave these results:

Moduli of a rectangular tube......	20.405.000	pounds
" " round tube............	24,500,000	"
" " elliptic................	24.300,000	"

Moduli of rails, experiments made by Morin:

Tredegar iron, double headed, maximum modulus..............................	27,730,000	pounds
Vignole's French rails. average modulus..	26,400,000	"
Dowlais rails, double headed minimum modulus........................	21,100,000	"

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* Consulting Engineer of the Cincinnati Southern Ry.

† November 1st, 1875.

‡ These experiments, which conclusively prove the superiority of the American system of bridge details, are very complete, and will, doubtless, attract much attention. There were

below 20 millions pounds.....	1	modulus
from 20 to 25 " "	7	moduli
" 25 to 30 " "	15	——
" 30 to 34.6 " "	5	——

the greatest difference being 31 per cent. Morin believed that the great variations of moduli (even of rails of same section and make) should be explained by the quality of the iron, and he judges that the better metal should show the higher modulus. But the great variations also of moduli of bars of undoubtedly excellent make and of great uniformity seem to disprove his judgment. He states that he has met with moduli, as low as 17,000,000 pounds, while the author has observed 18,000,000 as a minimum.*

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* General Morin acknowledges that it is pretty difficult to determine with exactness the *average value* of moduli to suit the results of old and new experiments, and he says: But, moreover, it must not be lost sight of that it happens *pretty often* that iron bars of the *same manufacture*, furnished by the *same* works, present *notable* differences in their resistance to flexure.

The distinguished French officer proposes a classification of iron of high grade (average modulus of 30,000,000 pounds), ordinary iron (modulus 25,000,000 pounds), and soft ductile iron (modulus from 21 to even 17,000,000 pounds).

But this classification can hardly be upheld, since the very best irons (for instance, Swedes, Russian, &c., brands) also are the softest and most ductile (and as regards ultimate strength somewhat weak) irons which, according to the classification, would have to belong to the *highest* and also to the *lowest* class. The experiments on eyebars, now parts of existing bridges, as made by the author, the iron being double refined (best-best) have given moduli from the lowest to the highest class.

There seems to exist this law—that the moduli of bars of same section made from double refined iron bars (rolled three times, packeted and welded twice), such as called *best-best*, are more uniform than bars made from best iron, such as were used by Herr Malberg in the Muhlheim suspension bridge. At least, many thousands of bars tested at Phœnixville, Pa., proved to be remarkably uniform in their moduli as long as they were of the same section, whilst the moduli were very variable when bars of different sections were compared. On the other hand, Styffe's experiments, which were made on excellent steel and iron, gave a maximum tensile modulus of 34,584,000, and a minimum of 27,585,000 pounds, which is similar to that of an iron rail from Avon, in Wales.

Morin's experiments on flexure of unhardened steel gave the following results:

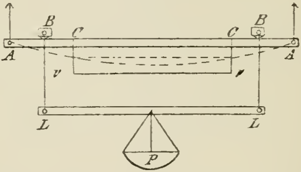
	Maximum.	Minimum.
From Petin & Gaudet, refined...	28,800,000	28,100,000
" " " puddled..	31,800,000	29,200,000
" " " crucible...	32,300,000	29,200,000
Krupp's	32,200,000	28,700,000
" mean of 17 experiments..	30,300,000	
English...	28,900,000	

It should be noticed, that even for the finest metal that we know, such as crucible

steel, the variation in its modulus amounted to 11 per cent., and for the renowned Krupp steel, 12 per cent.

The most accurate experiments are still to be mentioned (those of Herr Woehler), made by suspending the test-piece *A A*, so that it

Fig. 7.



could expand freely; the lever *L L* carried the load *P* and the arms *A B*, *A B* were exactly equal; the piece *B B*, acted upon by a constant moment of flexure, carried the measuring apparatus *C v*, *v C*, by which the deflection of *C C* could be very exactly found. In this apparatus the deflections of the test-pieces become comparatively very large, and piece *C C* was free from the influence of knife edges.

These are Herr Woehler's results:

Modulus of iron from	Laura huette......	28,930,000	pounds.
" "	Phœnix huette....	29,360,000	"
" "	Minerva huette. ..	31,680,000	"
"	Low Moor iron...............	31,230,000	"

Modulus of homogeneous iron from Pearson, Coleman & Co........	32,340,000	pounds.
" Bochum steel......	32,000,000	"
" Krupp steel................ ...	31,600,000	"

All these materials were of unusually excellent quality, and the maximum difference still was 12 per cent.*

The same variability which we have found to exist between iron and steel, not so much as to the quantity of carbon contained, as from imperfections of manufacture and other causes unknown to us, was noticed with the shearing or torsional moduli. Thus Duleau found for iron from Périgord, moduli of 14,450,000, and again of only 7,980,000 pounds. Iron from Arrièges gave 8,450,000, English Iron 9,860,000, and also 12,800,000

* Redlenbacher quotes the moduli of iron from 21,300,000 to 35,500,000 pounds. of steel from 28,500,000 to 34,100,000 lbs

Reuleaux (Der constructeur) for wire bars and ordinary steel gives 28,500,000, for cast steel (crucible steel) gives 42,700,000.

Kupffer, in St. Petersburg, by experiments on sound and flexure of small specimens, gets from 25,000,000 to 30,000,000 pounds.

Coulomb, Tredgold, Lagerhjielm and Wœhler found the modulus of hardened steel exactly equal to that of unhardened steel. Kupffer in some instances finds the modulus of hardened steel 6½ p. c. higher.

Styffe finds that the modulus of cold worked iron is low, but can be raised by exposure to a glowing heat. He also says that phosphorous lowers the modulus.

pounds. Wiebe in Berlin quotes the shearing moduli thus:

Soft wrought iron	9,000,000	pounds.
Bar iron.....	10,250,000	"
Steel......................	9,000,000	"
Finest cast steel	14,000,000	"

These figures prove that we cannot know the shearing modulus of any class of steel or iron without direct special experiment.

Our conclusions with reference to the supposition of a constant modulus of elasticity for the calculation of deflections and of continuous girders are—from known and unknown causes :—*First*, plain iron and steel bars vary in their moduli very considerably. The smallest modulus of iron was found to be 17,000,000, the maximum above 40,000,000 pounds. Single refined bars of same stock, manufacture and section vary in their moduli by 35 per cent. Double refined (*best-best*) bars vary little, as long as bars of same section are tested, but considerably with the sections, the minimum being 18,000,000, and the maximum above 40,000,000 pounds. The moduli of rails vary by 30 per cent., and similar results must be expected from common angles, beams, channels, &c. *Second*, consequently, riveted bridge members com-

posed of angles and plates of various thickness and manufacture, interrupted in their homogeneousness by punched holes, covering, reinforcing splice-plates, &c., must necessarily show still greater variations in their moduli than was found for plain integer bars. *Third*, the hypothesis of a constant modulus of elasticity of the material of a bridge being unfounded, the theory built on such hypothesis should be abandoned.*

Having arrived at such conclusion, we nevertheless must expect to hear an objection against its logical consequences, namely this —that numbers of continuous bridges do good service in practice. So they hitherto have done, not because the principle of continuity is admissible, but because the factor of safety used in their construction has hidden the error made in their design. For the same reason, the Victoria bridge in Canada stands, which is made continuous, but simply by

* Mr. Baker most pertinently remarks with reference to continuous girders: "The most expert mathematician would have to devote a month or more to the preliminary calculations of a very ordinary bridge, and the result deduced would not after all be more reliable in practice than those arrived at by comparatively simple modes of investigation, chiefly on account of the *varying elasticity of different portions of even the same plate of iron*."

combining two single spans whose greatest chord-sections are in their centres, whilst the greatest chord-strains, according to theory, would fall where the cross-sections are made the smallest. For the same reason, continuous draw bridges stand, which we find composed of two halves, each designed as a single span. The author knows of one instance, that a Hodgkinson cast iron beam was put in place upside down, so that the heavy tensional flange was under compression while the compressional flange of only one-fifth the area of the tensional one was strained under tension, and yet, on account of the factor of safety, the beam stood.

III.—Other Deficiencies of Continuous Girders, as regarding the imperfections of the theory, the danger from defective manufacture, from settling of piers, and the increase of strains by the action of the heat of the sun and the omission of the influence on the strains caused by deflections due to the web systems.

In order to find the exact extension or compression of a member of a bridge, we must know not only the modulus and the total strain of the member, but also its cross-sec-

tion. The problem of continuous girders, however, is to find this very section. The theory assumes that all sections are equal, or at least that the moment of inertia of a girder or a bridge is a constant throughout. Under this supposition we get *smaller* chord strains over the middle piers than exist in reality.

In the case of two equal continuous spans *under full load*, with uniform moment of inertia, we find the moment of flexure over a middle pier to be equal 0.125 l^2 p, where p represents the total load per lineal foot, and l denotes the length of each span in feet. But if we suppose that the same bridge, under full load, shall be strained equally per square inch, the co-efficient 0.125 becomes 0.1464, which indicates strains over the middle piers 15 per cent. higher. In reality, the continuous bridge being not perfectly varied in chord sections, the difference will be less ; but it may be remarked that the chords of a continuous bridge, properly designed according to specification, would only be about 10 per cent. lighter than those of equal single spans. With an enormous amount of labor, this deficiency of the ordinary theory can be corrected, and it has been done in a few

bridges. But considering the irregularity of the moduli, such labor seems superfluous.

A serious cause of errors in the construction of continuous girders refers to the distribution of strains over the posts and ties in case that two or more web systems have been adopted. In a single span bridge, a load brought on a panel joint of one separated web system, being split into two shearing forces in accordance with the law of the lever, there cannot be any mistake about the strain in a web member, as long as the end posts are vertical, and if they are inclined, the error can amount to only one increment of one panel load.

The problem of web strains with continuous girders depends not only on the law of the lever, but also on the angles of deflection α, β, γ, δ—not only of one, but of *all spans together*. We remember that by the moments M_1, M_2, M_3, &c., forces $\pm p_1$, $\pm p_2$, &c., were originated, which disturb the law of the lever. If, therefore, in a continuous bridge there are two or more web systems, we are utterly ignorant as to the distribution of the reactions over the two or more systems which, at every end pier and at every middle pier

are connected. How much of p_1, p_2, p_3, &c., is acting in one, and how much into the other system? This we do not and cannot know, for the distribution of the reactions will depend entirely on variations due to manufacture, in the mill as well as in the shops.* It may even happen that a member of one system receives tension and the other compression. It is therefore very desirable that continuous girders should be built with but one web system.

Hitherto in all our investigations we have made the supposition that the erection of continuous girders was of such perfection that the single spans were connected under the action of moments M_1 M_2, &c., which accorded completely with a perfect theory. Even with the best staging and under the supposition of the most careful workmanship it will be hard to perfectly fulfill this condition. But suppose it were possible, and that a pier set-

* In other words: Two or more systems of braces and fties in webs of single span bridges can be made *perfectly independent* of each other, and the strains in each, thereore, *can* be calculated by the law of the lever perfectly independent of each other, whereas in *each continuous* bridge with more than one web system these systems are connected together over the piers, therefore *never* are independent of each other, and can *not* be calculated separately.

tled. In this instance, the girder would receive considerable disturbances of its strains, which in some points would be decreased, while in others increased. The deeper the girder, the greater the disturbance from this cause would become, so that it seems advisable to leave to the girder as much plasticity as possible, by adopting a depth smaller than demanded by simple theoretical economy. In fact, this change in the value of calculated strains could become enormous; hence the piers of continuous girders should be built more substantially than is necessary for single spans. But this caution is costly.*

* It happens not unfrequently, that settling of piers of draw bridges causes difficulties in turning the superstructure. A bridge of this kind near New Haven, Connecticut, (Quinnipiat Bridge), was commenced two years ago, but is not yet in operation. The central pier tipped, the superstructure had to be jacked up, the masonry to be partly removed and newly laid. The calculations for the superstructure of this bridge had been made with great painstaking, involving much algebraic labor, which thus was most essentially vitiated by the nature of the substructure. Similar instances have happened elsewhere and where noticed, because the turning gear readily indicated the disturbance below. Of single span bridges being in their strains independent from the heights of support, we rarely hear of complaints caused by the settling of the piers.

The cost of bridge foundations and masonry differs between wide limits, according to quality, and for continuous bridges the very best class of either would be required.

The real economy of continuous girders, as claimed in Europe, when compared with single span lattice bridges, consisted in building the girders on land and then rolling them over the piers. This mode of erection is elegant, but it does not fully secure the fit of the superstructure to its bearings on the piers, and it is still doubtful whether this mode of erection always can compete with that of single spans, designed with the specific American details.† In the subsequent example of two 200 feet

† In the main building of the Philadelphia Exhibition the North Eastern R. R. of Switzerland has laid out a report on their bridges.

This official report is interesting in many ways. One remark thereof is; "*The erection of ironwork on scaffolds is preferred to the method of pushing the girders over the piers. This latter method never is allowed without intermediate temporary supports, and without reinforcement* (Armirung) *of the girders.*"

The acknowledged best builder in Switzerland always uses false works, but the mentioned French works push their continuous girders over the piers. However, it was observed that they thereby were likely to furnish second class work, and it happened that the violence, or the undue strains, connected with this method, caused rivet heads to fall off.

In case the method of pushing continuous girders over the piers is not to be used, their erection becomes *more* expensive than that of single spans. Not only that these scaffolds must be very unyielding and substantial, but *all spans* of the same set of continuous girders must be provided with false works at the same time, whereas, in the erection of single spans, only a scaffold for *one span* is used, or is usually used repeatedly. Also the risk, by erecting two or three spans at

continuous girders, we shall give figures as to the disturbance of strains in case the girders do not fit their supports.‡

What has been said as to the disturbances of strains by settlement of piers or by badly executed girders, is equally true in regard to the effect of the sun. Swing bridges have been drawn crooked by the rays of the sun falling upon one side. In others, the bottom chords are covered by floor timbers, and the top chords are considerably overheated by the sun, or unequally cooled under sharp winds. The effect of this unequal temperature is enormous, and it is sufficient (even) to raise a continuous bridge from a pier. In the case of tubular girders, this objection has peculiar force. Hereafter we shall take examples and calculate the strains caused by change of

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the same time, is considerably increased. The economical method of pushing girders over piers in a few rare instances, and under proper precautions (it was proposed for the Kentucky River Bridge, Cin. S. R. R.), might be used when the girders finally could be separated again by establishing hinges in alternate spans.

‡ In 1867 and later, the writer gave attention to the practical solution of the old idea of weighing the reactions of continuous girders, by means of hydraulic presses, and designed a cheap apparatus to accomplish this purpose. But the plan had previously been tried in the erection of a bridge in Silesia, Prussia.

temperature of the top and bottom chords of continuous girders.

The last objection urged against continuous girders, refers to the mode of proportioning those parts of their chords which at each passage of a train have to stand pressure as well as tension. The space for two spans thus strained equals 33 per cent. of the length of each chord. The European practice to proportion these parts is to find the maximum total strain, divide it by the maximum specified strain per square inch, and make the actual section as near to this theoretical section as can be done. This is radically wrong. Herr Woehler's experiments, perhaps the most thorough ever made, extending over a time of more than 12 years, have established beyond doubt, that the strain which controls the durability, equals the sum of the maximum tension and compression of a chord piece. A bar strained tensily to 35,200 pounds per square inch, can stand, say, 100,000,000 repetitions of such strains, but if at the same time strained compressively to 35,000 pounds, it will break after a small number of repetitions, say 100,000, whereas if strained to the limits of $\pm$ 17,600 pounds, it will show as much dur-

ability as if tensively strained to 35,200 pounds.*

Writers on continuous girders, generally erred in comparing girders of various systems of the same depth, whereas the proper depth of girders is a measure peculiar to each system of design and essentially depending on the relative quantities of chord and web-strains. The smaller the web-strains, the deeper a girder can be built. But the web material needed for continuous girders exceeds that of single-span girders by, say, 10 per cent., while the material necessary for the chords (according to theory) is just about as much less. The consequence is, that an increase of depth increases the web material more rapidly than is the case for single-span girders. Because of this, probably, parabolic girders were built much deeper in Europe than quadrangular trusses, and there is no reason why this same principle should be ap-

* For further information on this subject, compare Herr Woehler's Report in the *Berliner Zeitschrift fur Bauwesen.* These experiments were continued by Professor Spangenberg, of which a translation has appeared, published by Van Nostrand, New York. See also Inspecting Engineer Muller's article in the *Zeitschrift des Oestreichischen Ingenieur and Architecten Vereins*, 1873. See also Enbkam's *Zeitschrift*, 1875.

plied to continuous girders. Even an engineer like Prof. Kulmann in Zurich, made the mistake of comparing parabolic, continuous, quadrangules, single and Warren girders, by supposing all of them to be of the same depth, namely, one tenth of their lengths.

For comparison, we here give a few figures taken from the calculations for the new Buda-Pest bridge † in Hungary, now under progress of erection. There will be 4 spans, of 321 feet, carrying two tracks, depth 32 feet, each calculated for 3,000 pounds per foot ; strains 9,740 pounds per square inch ; compression correspondingly ; weight of parabolic trusses 285,500, and of continuous lattice trusses 270, 300 kilogrammes (two spans each). We know that American trusses of proper proportions can be built lighter and cheaper than parabolic trusses, and therefore, also, in this instance there is no reason for giving on the score of greater economy the preference to continuous lattice trusses. But the contractor had made a very low bid and desired the continuous bridge to be chosen, though originally, single spans were designed and bid upon. These continuous girders are intended to be

See Stummer's *Engineer*, Vienna, 1875.

rolled over the piers. The depth of one-tenth is decidedly too low for single spans ; it should have been taken at 40 instead of 32 feet.

Herr Schwedler, in Berlin, who certainly has as much experience in European bridge building as any other engineer, and who is so much an authority in theoretic matters that not even the most distinguished theorist can very well set him aside, in 1865* had made it his strict rule neither to build nor to recommend continuous girders or arches without at least hinges at the skewbacks. He builds a species of bow-string girders with depths of one-seventh of the span, which, though to American eyes complicated in details, yet are decidedly superior to continuous girders. In England far-going mathematical deductions on this topic have not been studied as much as in France, and in imitation of the French engineers in Germany. Nor has the distinguished late Professor Rankine dwelt on this subject very extensively. English

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* See Herr Schwedler's theses on bridge building in the *Zeitschrift fur Bauwesen.* The very excellent scientific pocketbook, *Des Ingenieurs Taschenbuch des Verein Hutte,* 10th edition, Berlin, does not treat continuous bridges.

engineers of name have expressed themselves that the expectations of those continental engineers were higher than could be realized in practice, that much more was to be done in advancing practical knowledge by means of well devised and well conducted experiments carefully, logically and rigidly interpreted, than by the application of hypotheses and mathematical reasonings, many of which simply concealed real ignorance, that there were few locations in which continuous girders were to be preferred, and, generally speaking, the circumstances were such that there would be no saving of money in their use.

Others have declared that the question of continuous trusses was too complicated for investigation, or that the formulae were too troublesome in application, &c., &c. This last objection, however, is only in part relevant. If the theory as given in text books, simplified as it now is by the theorem of the three moments were based on sufficiently correct hypothesis and were correct by itself, theoretically speaking, there would be no objection to the application in case of large bridges. But, unfortunately, the theory it-

self, if applied to skeleton trusses, is deficient.

This deficiency, due to the omission of the web system, is purely analytical, and yet has escaped the notice of the undoubtedly very able French mathematicians to whom we owe the development of this branch. The correct introduction of the consideration of the web system, *very variable* as it is, in *the unknown sections of its members*, would be a sheer *mathematical impossibility.*

The first application of the theory to iron structures probably was made in England. There were to be built shallow plate girders, whose web plates being of constant, or nearly constant thickness, participated in the resistance to the moments, and whose influence on the deflections was very small indeed. Here, then, one important objection to the common theory did not exist.

Moreover, there actually was a real economy in connection with the principle of continuity of *plate-girders.* For the web plates of these girders *practically* could not be reduced in thickness to the thereotical requirements, and web plates such as *practically* could be used for single span bridges were

also strong enough to bear the increased web strains of continuous girders. Moreover, the expense of such webs for large spans *compelled to small depths.* And since no material could be saved in the plate-webs, the theory of continuity offered a welcome help toward reduction at least of the chord material.

The modulus of elasticity once accepted to be a constant value, it was entirely rational and economical to use continuous plate-girders. On the contrary, it is theoretically wrong and practically not economical to build continuous skeleton structures. In other words, the huge English continuous plate-girders and their French imitations are more scientific than the extension of the ordinary theory to open-webbed trusses.

Strict logic led many French engineers to adopt constant sections of chords, so that *only* in case of *more than two* continuous spans by the mode of proper proportions as to their lengths real economy could be secured. And it also must be remarked that it likewise was a fully logical reasoning of these engineers to seek greater equaliaztion of moments by lowering the bearings on the middle

piers of continuous girders of two or more than two spans. Thereby pressure would be produced in the top chords over the middle piers, so that the great negative moments at these points (tension in top chords, pressure in bottom chords) would be reduced, and the smaller positive moments between the piers would be increased. This construction, theoretically, is rational, if the chord sections are made of constant sections ; but it becomes at once irrational and even more expensive if the chords are *varied.* Also, if the chords are varied, there is no longer any reason why the spans should not be equally long. And, in fact, it would be slightly more economical to arrange purposely for *great* negative moments over the middle piers.

In many treatises on continuous girders there exists great confusion as to alleged advantages of lowering of middle piers, and of best proportions of end and middle spans.

We have now explained why we cannot admit that it is desirable that American bridge engineers should seriously regard continuous girders, however attractive they may be to some mathematicians on account of the wide field for interesting problems presented, and

we shall proceed to briefly lay down the theory such as derived from the suppositions which were found questionable, whereupon an application shall be made to two 200 feet railway spans in comparison with single spans. There we shall find occasion to test all what has been said in previous paragraphs, also to examine the probable or possible errors of design, and thus to arrive at final conclusions.

This investigation will have a negative and also a positive value ; negative, because we spend useful time on the study of an objectionable system, and positive, because we once more will have occasion to learn that in our art also, the most perfect system must be the most simple one.

IV.—General Development of a simple Method of Finding the principal Formula of Continuous Girders.—The angles α, β, δ, γ, &c., can be found by considering one single formula developed in the theory of single span beams, which in the following paragraph will be used repeatedly. For the present, we make the same suppositions which are used by other writers, though contested by us. We assume therefore, *First.* The

modulus of elasticity of iron is a constant and known value. *Second.* The continuous girders throughout have the same cross-section, moreover they have straight parallel chords. *Third.* The deflections of these girders are not modified by the shearing forces; in other words, the struts and ties of the web do not change their lengths. *Fourth.* The web systems of the girders are so arranged that there is no doubt of the office of each separate system of struts and ties, which condition can only be fulfilled in case of but one system of diagonals and posts. *Fifth.* The temperature of all members is alike, and cannot change in any separate member. We use this notation: E is the modulus of elasticity in pounds per square inch, which, as known, is the measure of stiffness of material, the greater the modulus or the less the value of $\frac{1}{E}$ the less proportionally are the elastic deformations. I is the moment of inertia of the girder, equal for a skeleton truss, to the cross-section of one chord multiplied by one half the square of depth of the truss, all dimensions taken in inches. Like E, the value I stands in inverse geometrical propor-

tion to the deflection of a beam or girder. $p_1, p_2 \ldots p_n \pm {}_1$ denote the *elastic reactions* in pounds caused by the unknown moments

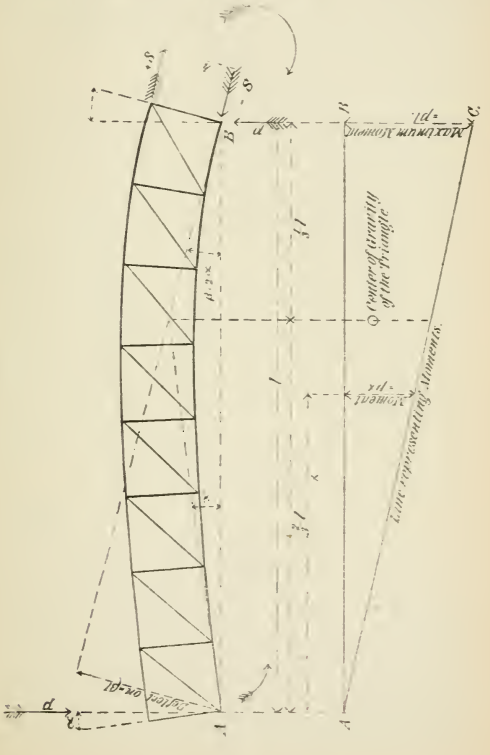


Fig. 8.

M_1 M_2 M_3 . . . M_{n1} over the middle piers of continuous girders. l_1 l_2 l_4 . . . i_n are the lengths of the spans in inches, consequently M_1 M_2 . . . M_{n-1} must be measured in pound inches.

The above figure represents a truss A B, which is supposed to be acted upon by no other forces but the pair $+ S_1 - S_1$, which create a moment $M = Sh$ counteracted by a force p in A. This force p in combination with $- p$ in B on the lever l (= A B) has the tendency to turn the truss A B in opposite direction to M, and to produce equilibrium; consequently pl must equal M. The sum of the horizontal as well as of the vertical forces being zero, no movement of the truss A B will be possible; nevertheless its elasticity will cause a flexure which increases in curvature from A to B. This is due to the moments of flexure increasing in geometrical progression from A to B, which moments in the triangle A B C are represented by the straight line A C. The maximum moment occurs at B and is $= M = Sh = pl$. For any distance x, from A the moment will be $Mx = px$.

The above figure also represents that the

total strains in the chords increase in geometrical proportion from A to B. At B the total strains will be S and $-S$, in A the strains will be zero. The chords being supposed to be equally strong in section, the strains per square inch likewise increase in a geometrical progression from A to B. The web strains, however, remain constant through the whole girder, because, according to the nature of this problem, the shearing force has a constant value $= p$.*

We know that the expressions for the angles α and β must contain E and I as divisors, and l and the maximum moment as multipliers, so that we only need find the coefficient to this expression. Actually the development gives:

$$\left.\begin{aligned} \alpha &= \frac{1}{6}\frac{M l}{E I} = \frac{P l^2}{6 E I} \\ \text{and} \\ \beta &= \frac{1}{3}\frac{M l}{E I} = \frac{P l^2}{3 E I} \end{aligned}\right\} (III.)$$

so that β is twice as great as α;)see Fig. 8).

In case the truss A B had been perfectly varied in sections to suit the moments, the

* In the sequel it will be shown how the formulæ for the angles α and B, can, under the suppositions made, be found without the aid of the infinitessimal calculus.

coefficient of β would no more be $\frac{1}{3}$ but would have increased to $\frac{1}{2}$, which is 50 per cent. more than under the supposition of a constant moment of inertia I, for any section of the truss $A\,B$; which result indicates need of caution in making this supposition for continuous girders.

The simple law contained in Eq. (III) is sufficient to easily solve the remainder of questions embodied in the theory of continuity. Suppose the girder $A\ B$ to be acted upon by this moment M_1 in A and by the moment M_2 in B, both moments acting towards an increase of upward flexure. What will be the angles α and β? This problem is only a corollary to the first. We have: end points A, B; moments at these, M_1, M_2:

Angles due to M_1 at $A = \dfrac{M_1\,l}{3\,E\,I}$ at B $\dfrac{M_1\,l}{6\,E\,I}$

" " M_2 " $\dfrac{M_2\,l}{6\,E\,I}$ " $\dfrac{M_2\,l}{3\,E\,I}$

Total angles.................. $\alpha = \dfrac{M_1\,l}{3\,E\,I} + \dfrac{M_2\,l}{6\,E\,I}$

$$\beta = \frac{M_1\,l}{6\,E\,I} + \frac{M_2\,l}{3\,E\,I}$$

In case M_1 were $= M_2$ there would be throughout the girder a constant moment

of flexure, and α would become equal to $\beta = \frac{M\,l}{2\,\mathrm{E\,I}}$. In this instance, the elastic line would be uniformly curved, and part of a circle whose radius is $\rho = \frac{E\,I}{M}$, as well known from the theory of single spans.

Finally, we have to determine the angles γ and δ of a single span exerted by a single panel load P. In Fig. 9, P represents the panel load at the distances a and b from points A and B, $a + b$ being equal to $A\,B = l$. By the law of the lever, the reaction of the pier at A will be $\frac{P\,b}{l}$ and at pier B, it will be $\frac{P\,a}{l}$. $E\,F$ representing the tangent on the elastic curve at D, the angles β and β_1, are known as well as the angles α and α_1. The angles $\varphi + \psi$ together must equal $\beta + \beta_1$. (Consider that $\varphi + \psi +$ angle $D = 180°$, and that $\beta + \beta_1 + D$ also $= 180°$). The angles of deflection being very small, can be considered as equal to their tangents, namely: $\varphi = \frac{d}{a}$; $\psi = \frac{d}{b}$; and $\varphi : \psi = b : a$. But on the other hand $\beta = \frac{M\,a}{3\,E\,I}$ and $\beta_1 = \frac{M\,b}{3\,E\,I}$ so

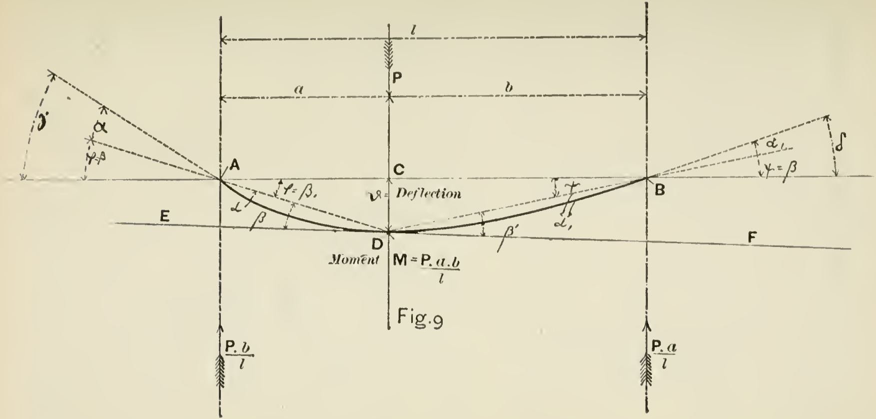

l
P
a
b
A
C
B
E
F
D
γ
α
$\varphi = \beta$
$\varphi = \beta_1$
α
β
ϑ = Deflection
β'
α_1
δ
Moment M = $\frac{P.a.b}{l}$
$\frac{P.b}{l}$
$\frac{P.a}{l}$

Fig. 9

that $\beta_1 : \beta = b : a$, and since $\varphi + \psi = \beta + \beta_1$ apparently $\beta_1 \quad \varphi$ and $\beta \quad \psi$, so that simply:

$$\left.\begin{aligned} \gamma &= \alpha + \varphi = \frac{M\,a}{6\,EI} + \frac{M\,b}{3\,EI} \text{ and} \\ \gamma &= \frac{P\,a\,b}{6\,l\,E\,I}\,(a + 2\,b) \quad \delta = \alpha_1 + \psi \\ &= \frac{M\,b}{6\,EI} + \frac{M\,a}{3\,EI} = \frac{P\,a\,b}{6\,l\,E\,I}\,(2\,a + b) \end{aligned}\right\} \quad (IV.)$$

In case the girder $A\ B$ should have carried any number of loads, $P_1\ P_2$ &c., with distances $a_1\ b_1$, $a_2\ b_2$, $a_3\ b_3$, &c., there would have been—

$$\gamma = \frac{1}{6\,l\,E\,I} \times \left(\begin{Bmatrix} \text{the sum of all} \\ \text{expressions} \end{Bmatrix} \right.$$

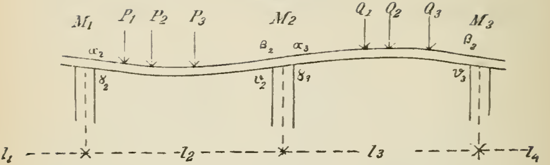
$$\left. P\,a\,b\,(a + 2\,b) = \frac{\Sigma\,P\,a\,b\,(a + 2\,b)}{6\,l\,E\,I} \right)$$

$$\nu = \frac{1}{6\,l\,E\,I} \times \left(\begin{Bmatrix} \text{the sum of all} \\ \text{expressions} \end{Bmatrix} \right.$$

$$\left. P\,a\,b\,(b + 2\,a) = \frac{\Sigma\,P\,a\,b\,(2\,a + b)}{6\,l\,E\,I} \right)$$

Now we are prepared to write the final formula of continuous girders. The equilibrum of the moments M_1, M_2 and M_3 with the forces P_1, P_2, &c., and Q_1, Q_2, Q_3, is found from (Eq. *I.*) $\delta_2 + \gamma_3 = \beta_2 + \alpha_3$ where

Fig. 10.



$\delta_2 + \gamma_3$ are the angles of deflection due to P_1, P_2, &c., Q_1, Q_2; and $\beta_2 + a_3$ are the angles of elevation due to M_1, M_2 and M_3—of the spans considered as single ones.

Their values are : $\delta_2 = \frac{1}{6\,E\,I\,l_2}\,\Sigma\,[P\,a\,b\,(2\,a + b)]$ for span l_2,

$\gamma_3 = \frac{1}{6\,E\,I\,l_3}\,\Sigma\,[Q\,a\,b\,(2\,b + a)]$ for span l_3.

$\beta_2 = \frac{M_1\,l_2}{6\,E\,I} + \frac{M_2\,l_2}{3\,E\,I};\ a_3 = \frac{M_3\,l_3}{6\,E\,I} + \frac{M_2\,l_3}{3\,E\,I};$

consequently

$$6\,E\,I\,(\delta_2 + \gamma_3) = M_1\,l_2 + 2\,M_2\,(l_2 + l_3) + M_3\,l_3 \quad (V.)$$

which actually is the equation of Henry Bertot ; also :

$$\frac{1}{l_2}\,\Sigma\,[P\,a\,b\,(a + 2\,b) + \frac{1}{l_3}\,\Sigma\,[Q\,a\,b\,(2\,a + b)] = M_1\,l_2 + 2\,M_2\,(l_2 + l_3) + M_3\,l_3 \quad (VI.)$$

This equation is of the first degree and contains three unknown quantities, viz.: M_1, M_2,

M_3, whilst the expression on the left side is fully known since the loads P_1, P_2, P_3, &c., and Q_1, Q_2, Q_3, with their distances a and b from the end points of each truss are given quantities.

At every central pier of a continuous girder there is an equation of this form, and there is also an unknown moment, so that we have as many equations of the first degree as there are unknown moments. The problem to find these moments consequently is solved analytically, though the labor of solving these equations algebraically, in case of many spans, is rather tedious.

If we introduce for a and b the corresponding number of panels, call $l = n\,d$, where d is the length of each panel, put $a = m\,d$ and $b = l - a = (n - m)\,d$, we arrive at these simplifications for the expressions on the left side of Eq. (VI);

instead of $\frac{1}{l}\ \Sigma\,[P\,a\,b\,(a + 2\,b)]$, we get

$$\frac{d^2}{n}\ \Sigma\,[Pm\,(n - m)\,(2\,n - m)]\ (A.)$$

instead of $\frac{1}{l}\ \Sigma\,[P\,a\,b\,(2\,a + b)]$, we get

$$\frac{d^2}{n}\ \Sigma\,[Pm\,(n^2 - m^2)]\ \ (B.)$$

The expressions A and B for the spans l_1 l_2 l_3 l_4, &c., l_n being denoted by A_1 B_1, A_2 B_2, A_3 B_3, A_4 B_4, &c., A_n B_n, Eq's ($VI.$), become ; (since the first and last moments M_0 and $M_n = O$

$$\left.\begin{array}{l} A_2 + B_1 = 2\, M_1\, (l_1 + l_2) + M_2\, l_2 \\ A_3 + B_2 = M_1\, l_2 + 2\, M_2\, (l_2 + l_3) + M_3\, l_3 \\ A_4 + B_3 = M_2\, l_3 + 2\, M_3\, (l_3 + l_4) + M_4\, l_4 \\ \quad \text{\&c.,} \qquad \text{\&c.,} \qquad \text{\&c.} \\ A_{n-1} + B_{n-1} = M_{n-2}\, L_{n-1} + 2\, M_{n-1}\, (l_{n-1} + l_n) \end{array}\right\} (VII.)$$

For two continuous spans, there is but one middle pier, and we have this equation only : $A_2 + B_1 = 2\, M_1\, (l_1 + l_2)$ and, in case that $l_1 = l_2$, finally

$$M_1 = \frac{A_2 + B_1}{4\, l} = \frac{d}{4\, n_2} \left| \Sigma \left\{ \begin{array}{c} Pm\,(n-m)\,(2n-m) \\ \text{for span II.} \end{array} \right\} \right.$$
$$+ \Sigma \left\{ \begin{array}{c} Pm\,(n^2-m^2) \\ \text{for span I.} \end{array} \right\} \Bigg|$$

The values M_1 M_2 M_3, &c., M_{n-1} being found; Eq's (II) teach how to calculate the *elastic reactions* p_1 p_2 p_3, &c., p_n, which in combination with the static reactions of each span due to the law of the lever give the actual reactions of the piers, that may be positive or negative, compression or tension.

The values E and I in Eqs. (V), (VI), and (VII), have totally disappeared, but the suppositions of their being constant throughout

the whole bridge *are embodied* in the equations, and without these suppositions being fulfilled the equations cease to be correct. In fact, E and I have only disappeared because we suppose them to be *constant* values, and should not have disappeared in reality.

Next we have to make corrections of these formulæ for the instances that the continuous girder does not properly fit to its bearings, on end or middle piers. Such misfits may arise from settling of the piers, from bad manufacture of the iron trusses, or from the effect of the rays of the sun being greater on one chord than on the other, or from winds cooling one chord sooner than the other. This investigation, which simply consists of a reapplication of the principle of continuity, will give us another opportunity to show how simply these problems can be solved with our method.

Suppose A B to be a straight line drawn through two end bearings of a continuous girder, and that d_1 d_2 d_3 denote the depressions or elevations of the middle piers, positive in case of elevation, negative in case of depression. Further, suppose the continuous girder to be cut into n single spans, freely placed on their supports. The problem then

is this, which additional or correctional moments M_1 M_2, &c. M_n are necessary to again connect the girders continuously?

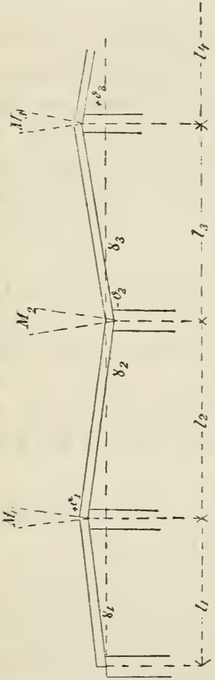


Fig. 11.

This problem at the first sight is nearly the same as that which we have solved. In the previous case, the moments M_1 M_2 M_3, had to lift up the single spans in such a manner as to make the sum of deflections $\delta_m + \gamma_m +_1 = \beta_m + \alpha_m +_1$. In this problem there are also angles δ and γ; but δ and γ of each span are equal in value, which in the problem just solved was not necessarily the case. Again, the angles δ and γ in the solved problem, were below the horizontal line drawn through the middle pier, whose equation (I) was under examination. In the

present problem, those angles may be above the horizontal line, consequently there may be cases when we shall have to consider them as negative.

There may arise instances that one or more of these moments will no longer draw together the top chords of the trusses, but push them apart; in other words, the moments M_1 M_2 may bring pressure in top chords, and tension in the bottom chords. We then have for our problem $\delta_m = \gamma_m = \pm \left(\frac{d_m - d_{m-1}}{l_m} \right)$;

the positive sign to be taken if the leg of the angle is below and the negative if the leg is above the horizontal line through the middle pier under consideration. In all other respects the problem is the same as the one we have just described; namely, this is the general equation:

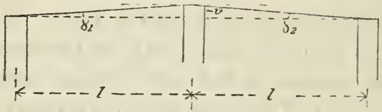
$$6\ E\ I\ (\gamma_m + \gamma_{m+1}) = M_{m-1}\, l_m + 2\ M_m$$
$$(l_m + l_{m+1}) + M_{m+1}\, l_{m+1}.$$

For n spans, there are $(n - 1)$ equations of this kind, and the moments M_0 and M_n are equal to zero. The values γ are to be substituted with their proper signs.

In the special instance of two equal spans

d, being an elevation of the middle pier above the line A, C, we have

Fig. 12.



$\gamma_1 = \gamma_2 = \delta_1 = \delta_2 = \frac{d}{l}$. Both angles are *below* the line A C, and consequently positive; we therefore have

$$4\ M = \frac{6\ E\ I}{l} \left\{ 2\ \frac{d}{l} \right\} \text{ and } M = \frac{3\ E\ I\ d}{l^2}$$

so that this elastic end reaction becomes

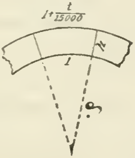
$$P = \frac{3\ E\ I\ d}{l^2}. \quad (VIII.)$$

If d had been negative, M would have been a moment, causing (above the middle pier) pressure in top and tension in bottom chord.

We now proceed to our last theoretical problem, namely, to calculate the influence of heat on one chord. Suppose, therefore, a properly manufactured girder resting on supports A B C D, etc., with the bottom chord covered by floor-planks: the top chord expands by the heat of the sun, the difference

of temperature between both chords being t degrees Fahr. In this instance, the uniformly heated top chord will expand $\frac{1}{150,000}$ of its length for each degree Fahr. If the girder is first considered to be without weight it must assume a flat arc, whose radius is easily found. Two posts which originally were

Fig. 13.



parallel have spread apart $\frac{150,000}{t}$ of the panel length, and consequently $1 : \rho : : \frac{t}{150,000} : h$ or $\rho = 150,000 \frac{h}{t}$. But the radius ρ being found, it is easy to also calculate the elevation of this flat circle above each middle pier, and this known the problem is at once reduced to the previous one.

Especially for two equal continuous spans there is an elevation of the girder equal to $\frac{l^2}{2\rho}$, which is the natural position of the girder

considered without gravity, the bed plate being placed $\frac{l^2}{2\rho}$ below the bottom chord; we have therefore $d = -\frac{l^2}{300,000}\frac{t}{h}$ and Eq's

(*VIII*) (in inches), $M = -\frac{E\,I}{1,200,000}\frac{t}{h}$ and
$p = -\frac{E\,I}{1,200,000}\frac{t}{l\,h}$, (*IX*);

where the minus sign indicates — regarding M, that the moment causes compression in the top, and tension in the bottom chord—and regarding p, that the end piers really are pressed by this elastic reaction; in other words, that p *increases* the pressure on the end piers as caused by dead and live loads on the girder. Dead and live loads, however, actually press down the girder to the middle pier either partly or wholly.

The moments of correction, M_1, M_2, M_3, &c., of a girder being found, for unequal positions of bed-plates as well as under consideration of heat in one of the chords, these results have to be represented on the diagram sheet of moments, shearing forces and strains, and be added algebraically to the moments and shearing forces due to the dead and live loads, when it will be seen whether these last

are sufficient or not to cause pressure always on the bed-plates.

Having now laid down the mathematical principles of the ordinary theory of continuity, it remains a mere mechanical labor to apply these principles and their resulting formulæ to any practical number of continuous spans, which labor may, however, require much patience. This theory is founded on the supposition that the angles of deflection and elevation γ, δ, α, β, are not influenced by the deformations due to the web systems, which assumption was about justified in the calculation of homogeneous plate girders such as we know to have been first used in England and France.

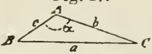
Is such simplification of theory also justified in case of continuous *trusses* of great depth?

It is impossible to investigate by direct analysis this cause of error. For we do not know the sections of the web members, nor can we consider them of equal value, nor could we estimate this unknown quantity even if we would assume it as a constant value, as was done with the unknown chords.

All that we could do would be: first to proportion a continuous bridge under consideration of the chords only, thereupon to calculate the correction due to the web and then make another calculation founded upon the corrected sections. In this manner with a great deal of labor we could finally succeed to proportion a continuous bridge properly. This labor indeed would be immense.

We now shall develop the necessary formulæ towards consideration of the deflections due to the web system of continous and other trusses.

Fig. 17.



Let $A\ B\ C$ be a triangle whose sides $a\ b\ c$ have been altered by very small quantities Δa, Δb, Δc; the problem is to find the alteration $\Delta \alpha$ of an angle. We have $a^2 = b^2 + c^2 - 2\ b\ c\ .\ \cos.\ \alpha$, which, by inserting the differences, leads to $(a + \Delta\ a)^2 = (b + \Delta\ b)^2 + (c + \Delta\ c)^2 - 2\ (b + \Delta\ b)\ (c + \Delta\ c) \cos.\ (\alpha + \Delta\ \alpha)$.

By developing this equation and considering that the squares of differences are very small quantities in comparison with their first powers, we get:

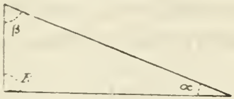
$a\,\Delta\,a = b\,\Delta\,b + c\,\Delta\,c - (b\,\Delta\,c + c\,\Delta\,b)$ $\cos.\ \alpha + b\,c \sin.\ \alpha\,\Delta\,\alpha$.

Hence we derive the value of $\Delta\,\alpha$.

$$\Delta\,\alpha = \frac{a\Delta a - b\Delta b - c\Delta c + (b\Delta c + c\Delta b)\cos.\ \alpha}{b\,c\,.\,\sin.\ \alpha}$$

By applying this formula to a rectangular triangle the formula is simplified into:

Fig. 18.



$$\left.\begin{aligned}\Delta\ \alpha &= \frac{\Delta\ a}{b} - \frac{\Delta\ d}{d}\cdot\frac{a}{b}\\ \Delta\ \beta &= \frac{\Delta\ b}{a} - \frac{\Delta\ d}{d}\cdot\frac{b}{a}\\ \Delta\ R &= -\frac{\Delta\ a}{b} - \frac{\Delta\ b}{a} + \frac{\Delta\ d}{a}\cdot\frac{d}{b}\end{aligned}\right\}(1.)$$

The sum of $\Delta\ \alpha + \Delta\ \beta + \Delta\ R = 0$, as expected.

These few formulæ (1) are sufficient to calculate the angles of deflection at a joint of a properly built skeleton bridge.

Fig. 19.

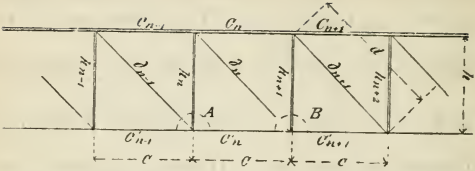


Fig. 19 represents part of a quadrangular truss, whose panels have the length, c, whose height is h, and whose diagonals are of the length d. The truss being under transverse strain, receives alterations of the lengths of its members and consequently of its angles. The angle A originally was 180 degrees, we now have to calculate the small alteration γ'_n of this angle. The alteration is the sum of the alterations of the three angles around A, namely:

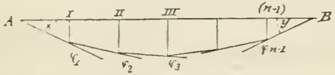
$$\left.\begin{aligned} \gamma_n = \frac{\Delta\, c_{n-1} - \Delta\, c^1{}_n}{h} + \frac{\Delta\, h_{n1} - \Delta\, h_n}{c} \\ - \left(\frac{\Delta\, d_{n-1} - \Delta\, d_n}{h\, c}\right) . d \end{aligned}\right\} \quad (2.)$$

In this expression $\dfrac{\Delta\, c_{n-1} - \Delta c^1{}_n}{h}$, representing the influence of the chords c and c^1, is a sum, because, if c_{n-1} is under compression, $c^1{}_n$ will be under tension and the abso-

lute values of the alterations of these quanti ties will add together. The influence of the posts $\frac{\varDelta h_{n-1} - \varDelta h_n}{c}$ and of the diagonals $(\varDelta d_{n-1} - \varDelta d_n) \frac{d}{h \cdot c}$ are actual differences of absolute numbers. Hence it follows that, generally speaking, the influence of the chords on the value of deflection must be more important than the influence of the web members.

The theory of the elastic line, such as developed with the integral calculus, throws off the influence on γ caused by the posts and diagonals, whilst only the chords are considered. Suppose the originally straight bottom chord of a beam has deflected and the angles of 180° at I, II, III. . . have altered by the values $\gamma_1, \gamma_2, \gamma_3, \ldots \gamma_{n-1}$, which alterations here are negative values.

Fig. 20.



The question arises which will be the angles x and y? The originally horizontal

chord pieces c_1 c_2 c_3 . . . will form angles with the line A B, as follows:

x, $(x + \gamma_1)$, $(x + \gamma_1 + \gamma_2)$, $(x + \gamma_1 + \gamma_2 + \gamma_3)$, . . . $x + \gamma_1 + \gamma_2 + \gamma_3 \ldots + \gamma_{n-1})$.

Here also exists the equation:

$$\left.\begin{array}{l} -(x+y)=(\gamma_1+\gamma_2+\gamma_3 \ldots +\gamma_{n-1}) \\ \textit{so that} -y=x+\gamma_1+\gamma_2+\gamma_3 \ldots +\gamma_{n-1}. \end{array}\right\} \quad (3.)$$

The chord pieces c_1 c_2 . . being equally long, the sum of the sines of the angles which are formed by c_1 c_2 . . . c_{n1}, with the horizontal line A B must be equal to zero. And since the angles are very small, their sines can be put equal to the angles themselves. Consequently we arrive at this equation:

$$0 = x + (x + \gamma_1) + (x + \gamma_1 + \gamma_2) + (x + \gamma_1 + \gamma_2 + \gamma^3) + \ldots + (x + \gamma_1 + \gamma_2 + \ldots + \gamma_n\, n-_1) \quad (4.)$$

or,

$$-n\,x = (n - 1)\,\gamma\,1 + (n - 2)\,\gamma_2 + (n - 3)\,\gamma_3 + \ldots + 2\,\gamma_{n-2} + \gamma_{n-1}.$$

and likewise we have: $-n\,y = (n - 1)\,\gamma_{n-1} + (n - 2)\,\gamma_{n-2} + (n - 3)\,\gamma_{n-3} + 2\,\gamma_2 + \gamma_1$.

In case of the span A B being uniformly loaded and supported at both ends the angles x and y would be equal, and since the sum

$x + y$ equals the negative sum of the angles $\gamma_1 + \gamma_2 + . \gamma_{n-1}$ each one would be half this sum. By inserting the values of equation (2) in the expression for $x + y$ of a uniformly loaded truss, all $\Delta\ h_n$ and $\Delta\ d_n$ will disappear, with exception of the influence of the end posts and of the end diagonals of each system. The chords, however, will remain in the formula for x and y under any circumstances. Of a uniformly loaded beam, resting upon two supports, the influence of the web on the angles x and y is as follows:

$$- x = - y \text{ due to web} = {}^1/_2$$

$$\left[\frac{\Delta\ h_o}{c} + \frac{\Delta\ h_n}{c} - (\Delta\ d_o + \Delta\ d_n)\ \frac{d}{h\ c}\right]$$

in which expression $\gamma\ h_o$ and $\gamma\ h_n$ are negative values.

The total expression is negative, so that the influence of the web increases the angles x and y.

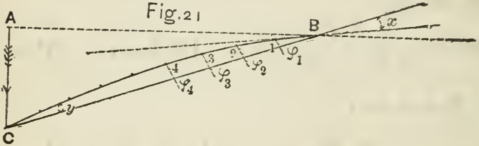
The web has a very considerable influence on the angle of deflection of a cantiliver beam. The originally straight beam $A\ B$ being fixed at B is bent into the curve B1, 2, 3, 4 C, when the angle x will be equal to

$$-\left[\frac{1}{n}\left((n-1)\gamma_1+(n-2)\gamma_2+\ldots 2\,\gamma_{n\cdot}+{}_2\gamma_{n-1}\right)\right.$$

$$\left.+\text{ angle } I\,B\,A\right]^*$$

In this sum no post and no diagonal of the truss $A\ B$ will disappear.

Fig. 21.



If we now suppose the special case of interest that the cantilever $A\ B$ in A, respectively C is acted upon by a single weight P, we know from the law of the lever that the chords are strained the more, the further they are from A. Hence we have to multiply P with its lever arm, from A to the chord piece in consideration, and to divide by the section of the member and by the modulus. Hence the value of compression or extension of a chord piece C_n equals $\frac{c.\ P.\ n.\ c.}{Section \times E.\ h}$ so that the

---

* Angle $I\ B\ A$ is the angle which the curve makes with $A\ B$ at B.

factor $\frac{c^2 P}{E \times section}$ is common to all extensions and compressions provided the sections are taken as constant, which, as well known, is one of the principal hypotheses of the ordinary theory of continuity. By inserting this expression into formula (4) w being the section, we get:

$$x = - \frac{2 c^2 P}{n E. w. h^2} \cdot \left[(n - 1)^2 + (n - 2)^2 + \ldots 9 + 4 + 1 \right]$$

$$x = - \frac{2 c^2 P}{n. E w h^2} \cdot \frac{(n - 1) . n . (2_n - 1)}{1 . 2 . 3}.$$

If n becomes a very large number $\frac{(n - 1) . (n) (2_n - 1)}{6}$ turns into $\frac{n^3}{3}$ and the angle $x = \frac{P . (c n)^2}{3 . E w h^2} = \frac{P . l^2}{3 . E . I}$ where $\frac{w h^2}{2}$ = the moment of inertia I, and $cn = l$ = the length of the span. This formula was one on which we based our method of the treatment of the common theory of continuous girders. Its use involves the supposition that the extensions of the diagonals, and the compressions of the posts, are immaterial in regard to the angles of deflection.

Since we know that the angles x and y equal the sum of all angles γ, y can readily be derived from x, and there will be

$$y = \frac{2\,c^2 \,.\, P}{E \,.\, w \,.\, h_2} \cdot \frac{n^2}{2} - \frac{2\,c^2 \,.\, P}{E \,.\, w \,.\, h^2} \cdot \frac{n^2}{3} - \frac{P \,.\, l^2}{6 \,.\, E \,.\, I \,.}$$

This was the other of the two equations *III.*

For the purpose of this paper it will be sufficient to explain the use of these formulæ on two equal continuous railroad spans, by which calculations we shall gain the opportunity to prove numerically the opinions laid down in previous paragraphs.

V.—Calculations of the Strains, Sections and Weights of Two 200 Feet Railroad Spans, compared under the same specification with a 200 feet single span. Examination of the question of economy.

Specification.—To construct two 200 feet, square through spans, 14 feet between trusses, of most economical height, with iron cross bearers, and with iron stringers 8 feet apart. For live load consider a train equal to 2,240 pounds per lineal foot, headed by a locomotive concentrating on a cross bearer $1^2/_3$ tons per lineal foot of a 16 foot panel. For late-

ral and transverse stiffness assume wind pressure of 25 pounds per square foot acting on the bridge when filled with passenger cars. Maximum direct strain in any point of the bridge to be 10,000 pounds—shearing strain 8,000 pounds—per square inch, and compressional sections of columns with flat ends to be multiplied by the factor $(1 + \frac{n^2}{5,000})$ where n represents the length of member measured by the least diameter of gyration of a mechanically well-built post; compression members with hinged joints to be treated correspondingly, according to theory. The connections of web diagonals and chords to correspond with the supposition of the calculation.

Under this specification, we divide the 200 feet spans into 12 panels of 16 feet 8 inches long, and we assume the dead weight per lineal foot equal to 1,200 pounds.

First.—Calculations of a continuous bridge of two spans, 200 feet each.

In accordance with the specification of one truss the

panel live load is $\frac{2.240}{2} \times \frac{200}{12} = 18,666$ lbs.;

$$\text{increment, } \frac{18.666}{12} = 1{,}555 \text{ lbs.};$$

$$\text{panel dead load, } \frac{1.200 \quad 200}{24} = 10{,}000 \text{ lbs.};$$

$$\text{increment, } \frac{10.000}{12} = 833 \text{ lbs.};$$

$$\text{ocomotive excess, } \frac{2}{3} \times 18{,}666 = 12{,}444 \text{ lbs.};$$

$$\text{increment, } \frac{12444}{12} = 1{,}036 \text{ lbs.}$$

In the equation for moment over the middle pier (*Eq. VII*) the following values are to be substituted, for P, 18,666, 10,000 and 12,444; for n, 12; for m, 1, 2, 3, &c., to 11; for d, $\frac{200}{12} = \frac{100}{6}$ and for l, 200.

The elastic reactions p must be subtracted from the static reactions $P\frac{(n-m)}{n}$, and be added to the reactions of the middle pier $\frac{Pm}{n}$. In the table, calculation of the values p, $P\frac{n-m}{n}$ and $P\frac{m}{n}$ is carried out for a panel dead load, a panel live load, and a panel locomotive excess placed successively on the joints 1, 2, 3, &c., 11, of one span. The combinations of these values for both spans lead

to the maxima reactions over end piers (A), and over middle piers (V).* They also form all material necessary to calculate the maxima moments, not only of M over the middle pier, but at any other vertical sections of the girders.

We will next calculate the maxima moments:

(a) Moments due to dead load. $A =$ 40,170;

$M_1 = 2 \times - 7,415 \times 200 = - 2,966,000$ pound feet. Any moment M_m, is found by considering the m panel loads acting at their joints. There is namely, in accordance with the law of the lever:

$M_m = 40,170 \times m\, d - (1 + 2 + 3 +$ &c., $+ (m - 1) \Big)$ d $\times$ 1.0000.

or, $M_m = [40,170 - (m - 1)\, 5,000]\, m. \dfrac{200}{12}$

The value of this formula can be easily measured on the diagram of forces. (See Plate.)

(b) Curve of moments due to full live loads.

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* Simply on the law of super position of effects, following directly from the law ut extensio sic vis; also see Annales des Ponts et chaussees, 1874, paper on continuous girders by M. Choron.

Here is $A = 74.866$, $V = 130,466$, $p = 27,800$ pounds, and $M_1 = 200 \times 27,800 = 5,500,060$ pound feet. Any moment $M_m = [74,866 - (m-1)\,9,333]\,m.\frac{200}{12}$. (See Plate.)

(c) Curve of moments due to live load on one span only. $A = 88,766$, $p = 13,900$, $M_1 = 2,780,000$ pound feet. $M_m = [88,766 - (m - 1)\,9,333]\,m.\,\frac{200}{12}$. (See Plate.) The curves thus obtained enable us to find the maximum moment for any panel of the bridge. This is done on the diagram by adding the positive and negative moments occurring at any points. The moments M_a, M_b, M_c, can also be obtained by calculating or drawing, first, the curve of moments under the consideration of single spans, and then the (straight) line of moments due to the elastic reaction p, whereupon the difference of these values, for any point m, agrees with the values calculated as above.

The maxima shearing forces are now to be calculated:

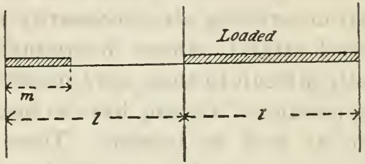
(a.) Shearing forces due to the dead load. These forces can be easily obtained by subtracting 1, 2, 3, &c. (n—1), panel dead loads

REACTIONS.

Panel Number = m.	Panel Dead Load = 10 000 Pounds.						Panel Live Load = 18 666 Pounds						Locomotive Excess $31\ 111 - 18\ 166 = 12\ 144$	
	A.			V.			A.			V.				
	Static Load. $P\frac{n-m}{n}$	$-p$.	A. Shearing Up.	Static Load. $P\frac{m}{n}$	$+p$	V. Shearing Down.	Static Load. $P\frac{n-m}{n}$	$-p$.	A	Static Load. $P\frac{m}{n}$	$+p$	V.	A.	V.
1	9 166	206	8 960	833	206	1 039	17 111	386	16 725	1 555	386	1 941	11 150	1 294
2	8 333	403	7 930	1 666	403	2 069	15 555	756	14 799	3 111	756	3 867	9 866	2 578
3	7 500	583	6 917	2 500	583	3 083	14 000	1 094	12 906	4 666	1 094	5 760	8 604	3 840
4	6 666	737	5 929	3 333	737	4 070	12 444	1 382	11 062	6 222	1 382	7 604	7 375	5 069
5	5 834	859	4 975	4 166	859	5 025	10 888	1 607	9 281	7 777	1 607	9 384	6 187	6 256
6	5 000	933	4 067	5 000	933	5 933	9 333	1 749	7 584	9 333	1 749	11 082	5 056	7 388
7	4 166	958	3 208	5 834	958	6 792	7 777	1 796	5 981	10 888	1 796	12 684	3 987	8 456
8	3 333	921	2 412	6 666	921	7 587	6 222	1 728	4 494	12 444	1 728	14 172	2 996	9 448
9	2 500	816	1 684	7 500	816	8 316	4 666	1 531	3 135	14 000	1 531	15 531	2 090	10 354
10	1 666	634	1 032	8 333	634	8 967	3 111	1 188	1 923	15 555	1 188	16 743	1 282	11 163
11	833	364	469	9 166	364	9 530	1 555	683	872	17 111	683	17 794	581	11 862
Sums	55 000	—7415	47 585	55 000	+7 415	62 415	102 666	—13 900	88 766	102 666	+13 900	116 566	....	....
Reactions one span loaded A.........			47 585	....	V =	62 415	....	A =	88 766	....	V =	116 566	....	....
Other span also loaded............			—7 415	....	....	+7 415	....	....	-13 900	....		+13 900	....	....
		A =	40 170	....	V =	69 830	....	A =	74 866	....	V =	130 466	....	....

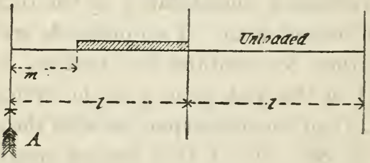
from the value $A = 40{,}170$. This operation is represented on the diagram of shearing forces.

Fig. 14.



(*b*.) Maxima live shearing forces acting in the direction of A. For this purpose, the second span is supposed to be unloaded, and a train to extend from the panel m to the middle pier.

Fig. 15.



(*c*.) Maxima live shearing forces acting upward in the center. For the calculation of these forces the second span is supposed to be loaded, and the first span also loaded from the end pier to panel m.

The bridge of which we are calculating the

forces is shown with two web systems. It has been explained* why it is impossible to calculate exactly, the strains occurring in each one of these systems. It must now be added that this uncertainty also necessarily attends the chord strains, whose determination is especially difficult in those cord pieces which, at each passage of a train, have to bear compression as well as tension. There is no method to overcome this imperfection of the theory. In the following calculation we have separated the systems, and have supposed that each system would act independently.

For the calculation of the forces V_c there arises another difficulty. The value p namely, is inflenced considerably by the total load on the second span. Two methods are possible, either to combine the system, 1, 3, 5, &c., 11 of the first span, with the system 1, 3, 5, &c., 11 of the other span, or with the system 2, 4, 6, &c., 10 of this second span. We have assumed, that instead of $\frac{13.900}{2} = 6{,}950$ somewhat more, namely, 8,000 pounds would be the value of p (due to the second span) for each of the systems of the partly loaded span, Fig. 14.

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* Page 47.

The following tables exhibit the results of the calculation, which is made simply by adding the respective values taken from the preceding table.

Panel.	Dead Load A.	Dead Load V.	Live Load. A.	Live Load. V.	Excess of Locomotive. A.	Excess of Locomotive. V.	Shearing Maxima. A.	Shearing Maxima. V.
System 0—1—3—5—7—9—11—12.								
0—1	8 960	1 039	16 725	1 941	11 150	1 294	82 556	....
1—3	6 917	3 083	12 906	5 760	8 604	3 840	53 275	......
3—5	4 975	5 025	9 281	9 384	6 187	6 256	27 962	17 033
5—7	3 208	6 792	5 981	12 684	3 987	8 456	6 481	38 833
7—9	1 684	8 316	3 135	15 531	2 090	10 354	....	63 717
9—11	469	9 530	872	17 794	581	11 862	....	91 146
11—12	....	....	....	....	....	....	....	120 448
	26 213	33 785	48 900	63 094	32 600	42 062	....	....
System 0--2--4--6--8--10--12								
0—2	7 930	2 069	14 799	3 867	9 866	2 578	67 391	....
2—4	5 929	4 070	11 062	7 604	7 375	5 069	40 101	6 778
4—6	4 067	5 933	7 584	11 082	5 056	7 388	16 723	26 873
6—8	2 412	7 587	4 494	14 172	2 996	9 448	....	50 274
8—10	1 032	8 967	1 923	16 743	1 282	11 162	....	76 506
10—12	....	....	... .	....	....	....	....	104 965
	21 370	28 626	39 862	53 468	26 575	35 645	35 645	....

The shearing maxima A and V are now to be combined, so as to obtain the total maxima in each panel, such as represented on the diagram of forces. For the calculation of the diagonal and post strains, the two systems again must be treated separately.

The diagrams for the chord and web strains in combination with the two tables referring to the web strains of each separate system, can now be used to calculate in the usual manner the members of the proposed bridge. For this purpose we choose a height of 25 feet (one-eighth of the span) and after consideration of the web strains in the end panels we arrive at the diagram of strains reprepresented. An examination of these strains will give proof that the chord strains cannot be properly determined without consideration of the diagonals, and that consequently the mere theoretical comparison of curves of moments and shearing forces may lead to considerable errors.

Top and bottom chords of continuous girders after all, cannot be calculated with perfect certainty, even under the objectionable suppositions made. It is, therefore, advisable to construct them to resist tension as well

as pressure. A proper section for these chords would be two built channels connected with lattice bars at top and bottom. The pins can be put with mechanical correctness through the centre lines of these channels, and the re-enforcements can be placed so that the pins bear against the metal added to the web plates. We construct the diagonals of weldless eye bars, and the counter rods with swivels. This arrangement has a scientific advantage. Each web member carries but one kind of strain; whereas, in bridges with diagonal web members only, diagonals, at least near the centre of the span, have to resist tension as well as pressure, and therefore must be designed to sustain the sum of both. Moreover, vertical posts are more convenient, with reference to the construction of the joints.

The built chord channels are calculated to be 16 inches deep, and the angle bars 3x3 inches, the latticing to be double top and bottom. The posts are also designed on this basis, with 2 rolled channels and latticing; their bearings are made flat against the bottom and top chords; the radii of gyration have been duly calculated.

The strains upon compression members are in exact agreement with the formula, the factor $\left(1 \times \frac{n^2}{5{,}000}\right)$ increasing from 1.12 of chords to 1.82 of posts; the section of the lightest post is taken at 8 square inches.

On this basis the sections of chords, diagonals, posts, counters, &c., have been determined in agreement with the specification. From the strain sheet thus obtained (see Plate) this bill of materials is calculated:

Chords, latticing, joint and reinforcing plates	85 912	pounds.
Posts with latticing, top and bottom bearings, rivets	35 560	"
Diagonals and swivels	43 292	"
Pins and rollers with cages	3 850	"
Cross-beams, hangers and washer-plates	16 000	"
Stringers,	25 600	"
Struts and portals	6 000	"
Lateral rods	4 000	"
Castings (end post feet and heads bed-plates, &c.)	5 000	"
Floor bolts and washers	3 000	"
Total weight of one iron 200 feet span	228 214	"

Iron, per lineal foot	1.141	pounds.
Timber and rails	300	"
Total dead load, per foot	1.441	"
Assumed weight per foot	1.200	"

A too light dead load, therefore, was assumed ; but the error amounts to less than

5 per cent. on the truss weights proper, say about 5,000 pounds in the span. The corrected weight of the iron-work of this continuous bridge would then amount to 1,166 pounds per lineal foot.

For the sake of comparison under precisely the same specification, for the same form of truss, for the same details and the same number of panels, a 200 foot single span has been calculated. This is the strain sheet with data and mode of computation ; *

Span 200 feet ; 12 panels, 16 feet 8 inches long ; diagonals for 27 feet depth, 31¾ and 43 feet ; secants, 1.17 and 1.59 ; tangents, 0.61 and 1.23 ; dead load, $1{,}200 \times \frac{200}{24} =$ 10,000 ; live load, $2{,}240 \times \frac{200}{24} = 18{,}666$ pounds per panel ; excess of locomotive load on a joint, 12, 444

Chords.--$3 \times 0.61 \times 28{,}666 = 53{,}000$; $2 \times 1.23, \times 28{,}666 = 70{,}600$;

$1 \times 1.23 \times 28{,}666 = 35{,}300$; $2\frac{1}{2} \times 1.23 \times 28\ 666 = 87{,}000$;

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* This example will show the wide difference as to the time required for the calculation of the strains of a single span bridge compared with continuous bridges.

Maxima Shearing Forces.

System.	1.	3.	5.	7.	9.	11.	12.
From dead load..............	30,000	20,000	10,000	....	−10,000	—20,000	—30,000
" live "	....	—1,555	—6,222	—14,000	—25,000	—38,888	—56,000
" locomotive excess........	....	−1,037	—3,111	—5,185	−7,259	− 9,333	—11,400
Maxima	....	....	....	19,185	42,259	68,221	97,400
Diogonals	....	....	....	30,000	67,000	109,000	114,000

System.	2.	4.	6.	8.	10.	12.
From dead load........................	25,000	15,000	5,000	−5,000	− 15,000	—25,000
" live "	....	—3,111	− 9,333	—18,666	−31,110	—16,000
" locomotive........................	....	− 2,074	− 4,148	−6,222	—8,296	—10,370
Maxima	....	....	8,481	29,888	54,400	81,370
Diagonals........................	....	....	14,000	48,000	86,000	130,000

$1\frac{1}{2} \times 1.23 \times 28,666 \times 53,000$; $\frac{1}{2} \times 1.23 \times$ 28, 600 = 17,600.

Addition.—53,000, 87,000, 70,600, 53,000 35,300, 17,600 ; whence the chord strains, 53,000, 140,000, 210,600, 263,600, 299,000, 317,000 pounds.

The compression members are designed—first, as done with the continuous span ; and second, to consist of hollow segment columns. The same sections are adopted for both cases, but with the hollow posts we gain all latticing and still have a greater factor of safety in regard to ultimate strength.

Bill of Materials.

Top chords	49,500	pounds.
Bottom chords	28,000	"
Pins and rollers	4,500	"
Posts	34,000	"
Diagonals	37,000	"
Cross bearers, &c	16,000	"
Stringers	25,600	"
Lateral struts and portals	6,000	"
Lateral rods	4,000	"
Castings	5,000	"
Floor bolts	3,000	"
Totol weight of iron	212,600	"
Weight per foot	1,063	"
Timber and rails	300	"
Total weight per foot	1,363	"
Weight assumed	1,200	"

Hollow colum chords	39,000	pounds.
Bottom chords	28,000	"
Pins and rollers	5,000	"
Hollow posts	27,300	"
Diagonals	37,000	"
Cross bearers	16,000	"
Stringers	25,600	"
Lateral struts and portals	6,000	"
Lateral rods	4,000	"
Castings	12,400	"
Floor bolts	3,000	"
Total weight of iron	203,300	"
Weight per foot	1,017	"
Timber and rails	300	"
Total weight per foot	1,317	"
Weight assumed	1,200	"

The weight assumed, 1,200 pounds, consequently was too light also for a single span, and the truss weight should be increased by 4 per cent., so that the actual weights would be respectively 1383 and 1337 pounds per foot. These still are respectively 58 and 104 pounds less than we obtained for the continuous girders.

Having now seen that in the construction of two continuous spans there is no economy, if compared with properly designed single spans, it will be well to examine the weights in detail.

These following, are the percentages of

weights as calculated for a supposed dead load of 1,200 pounds per foot.

	Continuous Girders. 25 feet deep.	Single Spans 27 Feet Deep.	
		Latticed Posts.	Hollow Columns.
Chords	37.7	36.4	33.
Webs..........	34 6	33.3	31.6
Both	72.3	69.7	64.6
Balance........	27.8	30.3	35.4

This comparison shows that the more perfect the detail design, the smaller the percentage of weight taken up by the chords and webs. The single span, with hollow wrought iron segment columns, gives the best result. The single span, with latticed posts, is superior to the continuous girders with latticed posts and chords, for the chords and webs still contain 2.6 per cent. less of the total weight in the first design than in the second. This is not alone due to the height, 27 feet, of the single span, for an increase in height would hardly reduce the chords of the continuous

girder, since $^5/_{12}$ of these chords cannot be reduced in section without lowering the heights of chord members, and therewith reducing the admissible chord pressure.

The panel length being taken at 16 feet 9 inches, the truss height could be increased to 33 feet without losing weight in diagonals, but the posts would become considerably heavier.

The maximum height of a truss is reached if an increase in height causes an increase of total weight ; that is, if by an increase of height the web and lateral bracing increases more than the chords decrease.

The most perfect compressional members permit the use of the greatest depth, since the weight of the posts is a large part of the total weights. The single span, with hollow wrought iron segment posts (Phœnix columns) therefore, has the smallest dead weight. From the variability of strains in their chords and webs, continuous girders require continuous riveted chords. Under this construction, loss of material seems unavoidable, because these members cannot be made without it, in practically too small sections at the points where the moments became zero. On the

other hand, it will be found advisable in continuous girders to avoid too great a variety of riveted members intermixed with eyebars. This construction has been tried several times in this country with drawbridges, but it is doubtful whether any gain actually is obtained by such design.

The continuous girders, such as here designed, have one advantage over the fixed span, because the posts have been arranged with two flat ends, whereas the single span was designed with posts of but one flat bearing.

	Continuous Girder.	Single Span; Latticed Members.	Single Span; Hollow Columns.
Posts, pounds.........	35,560	34,000	27,300
Diagonals, "	43,292	37,000	37,000
	78,852	71,000	64,300
Ratio	1.23	1.11	1.00
Chords—Pounds... ..	86,000	78,000	74,000*
Ratio	1.16	1.05	1.00

* With castings.

The foregoing comparison shows that the web of the best designed single span is 23 per cent. lighter than the web of the continuous girder. Theoretically (compare strain sheets) this advantage of the single span amounted to only 12 per cent.

Theoretically the chords compare thus: continuous girder 4,403, to single span 5,026, or as 1.00 to 1.14. In other words, for the same height of trusses, 27 feet, though the continuous girder appears theoretically to save 14 per cent. in the chords, in reality it causes a loss. While a continous girder of three spans, proportioned according to theory, would show a gain in the chords of 33 per cent., in fact (see Laisle and Schubler) executed examples of acknowledged excellence of design gave only 15 to 20 per cent. and this gain is only comparative since obtained under sacrifice of height, the depth being one twelfth instead of one eighth the average length of the three continuous spans.

Having given the practical figures and weights for 200 feet spans, it yet remains to show whether there actually was *any theoretical advantage* in favor of continuous skeleton structures. To this end, from the foregoing

tables, the *theoretical quantities* were calculated, consisting of the products of the length of each member into its maximum strain, respectively into the sum of positive, *plus* negative maximum strains (in case a member has to carry tension as well as pressure). In order also to show that *three* spans, of what in books is usually claimed as a more economical arrangement of length of spans, do not give any greater advantage than two continuous spans, a bridge of 600 feet total length has been calculated. Its outer spans have 11 panels of 16 8″ each, its middle span has 14 panels of 16′ 8″, so that the same panel length is considered which we assumed in the previous examples. This example will give evidence that Laisle end Schubler are correct in stating that two continuous spans are about as economical as three, and consequently also as economical as an arrangement of more than three span; in other words, that it is sufficient to confine our calculations to two continuous spans.

We get the following theoretical quantities in pound-feet, and hence by multiplying with $\frac{10}{3}$, and dividing by the unit strain of 10,000

pounds per square inch, also the theoretical weight of the trusses.

THEORETICAL QUANTITIES.

	Pound, Feet.	Total.	Weight per lin. 1' of trusses only.	
			lbs	Per cent
Single Spans..............Chords	837,670	1,555,340	519	100
27' deep (not deep enough).Webs	718,670			
Two continuous spans.....Chords	790,500	1,563,330	521	100.5
25' deep.....................Webs	772,830			
Three continuous spans....Chords	819,000	1,708,000	570	109.7
25' deep (a little too deep)..Webs	889,000			

It might be rejoined that the advantages of continuity better present themselves in case of heavier dead loads. Therefore, under precisely the same conditions, but for a dead load of 2,400 lbs., we have calculated two single spans, and three continuous spans. These are the

(N. B.—In the three continuous spans the effect of the heavy locomotive is not yet considered.)

THEORETICAL QUANTITIES.

		Pound, Feet.	Total.	Weight per foot of trusses. lbs	Per cent
Single Spans, 200', 27' deep .... Locomotive considered	Chords	1,130,000	2,045,000	682	101.4
	Webs	915,000			
Three continuous spans 25' deep .... Locomotive *not* considered	Chords	958,400	2,016,609	672	100.0
	Webs	1,058,200			

In this instance the continuous girders are designed too deep, and the single spans too shallow, for their proper heights the quantities of the chords should have become nearly equal to those in the webs ; and by also considering the locomotive load the theoretical advantage of 1.4 per cent. would have been turned the other way.

In all these examples, the *chords* of the continuous girders can be noticed to be *lighter* than those of single spans, whilst, reversedly, the *webs* of continuous girder are *heavier* than those of single spans, in such proportions

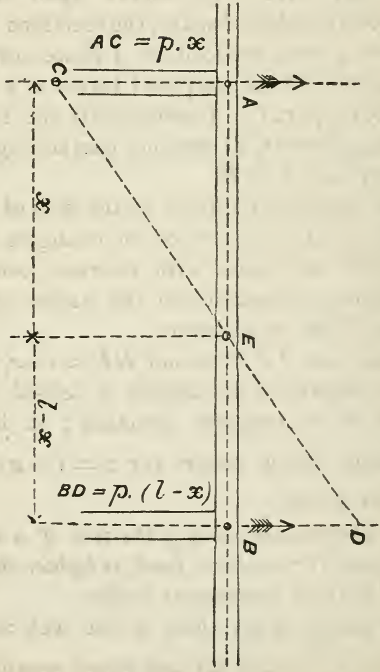
that the *gain in the chords is just about neutralized by the loss in the webs.*

It must be expressly stated that, if it were possible with continuous girders to save so much in the chords, that this saving, less the extra weight in the webs, would leave a final saving ; this would *only* indicate a saving in the theoretical value of the trusses. The connecting parts, as latticing, rivets, reinforcing plates, then the lateral strutting, lateral diagonals, and the whole floors remain constant quantities unaltered by the principle of continuity. In our example, about one-third of the iron of the whole bridge is a constant quantity, and a theoretical saving in the trusses of three per cent. could only realize two per cent. on the iron work of the whole bridge. Most writers on continuity, however, only mention the *theoretical* saving in the *chords*, without substracting the loss in webs, or without considering the quantity of constant weight of iron.

The reason why the webs of continuous bridges must become heavier than those of single spans, can easily be demonstrated by the following examination of a uniformly and fully loaded continuous bridge :

Be $A\ B$ a span of a continuous brige, uniformly and fully loaded by p pounds per lineal foot, $C\ D$ may represent the line of shearing forces, $A\ C$ being the reaction on

Fig. 21.



A, equal to px, and $B\ D$ being the other reaction, $= (l - x)\ p$.

It is well known that of a continuous bridge, even if uniformly loaded (for instance by dead load), save in the middle span in case of an odd number of spans, the reactions A and B arising from the load on $A\ B$ are *not equal* (on account of the couple of forces $+ \gamma - \gamma$, see above, part I). Consequently the line of shearing forces $C\ D$ does not pass through the centre point of $A\ B$.

The quantity required in the web of span $A\ B$ is equal to the sum of the triangles $A C E + E B D$, multiplied with a certain constant co-efficient dependant on the nature of the design of the web system.

Since then $A\ E = x$ and $BE = l—x$, there will be the web = co-efficient $\times\ [x^2+(l—x)^2]$.

This is no constant function; it has a minimum which occurs for $x = l - x = \frac{l}{2}$ in other words:

The *theoretical value of the web of a single span, even for uniform load, is lighter than it would be for a continuous bridge.*

For single spans there is the web = co-efficient $\times \frac{l^2}{2}$, and for two equal continuous

spans each one is = co-efficient $\times$ 0.531, l^2, this being 6.2 per cent. more than for single spans. *Practically*, in case of many spans the webbing of continuous bridges would only be slightly greater than for single spans; where it not that the *movable* load influences continuous girders very materially more than it does single spans. This is due to the fact that the point E in the previous figure, on account of the variable and moving live load, moves over a considerably greater part and further from the center of the span AB, than happens for single spans (compare strain sheet, pages 94 and 99).

Now, taken as granted that the theoretical web material of continuous bridges is greater than that for single spans, it follows directly that single spans can always be designed, which require not more total material than that of a continuous bridge. For it is known that, *theoretically* speaking, in trusses with parallel chords the least web material is independent of the height of truss. Consequently, even if in both systems, the theoretical web material would be alike, all that would be necessary would be simply to make the single span trusses correspondingly deeper.

Since, however, the webs of continuous girders require more material than single spans ; there is only a want for a *slight* increase of height of truss of single span in order to bring it on the same footing with the continuous one. Practically the total minimum is about reached if the web material and the chord material are equally heavy, and this limit happens therefore sooner for continuous than for single span trusses.

Hitherto we have developed the strains, sections and weights of two continuous railroad girders of 200 feet length. We have made the incorrect supposition that the moment of inertia of these girders is a constant value. But in reality the effective sections of the parallel chords vary from 16 to 38 square inches. Therefore, the curves of moments and the values of shearing strains given by our formulæ, and on which we based the estimate of weight, are not correct. We have stated that the maximum moment over the middle pier would be 15 per cent. greater, were the moment of the inertia of the girders so varied as to produce an equal maximum strain of 10,000 pounds per square inch of the bridge totally loaded, and this difference in

continuous girders with three openings sinks to seven per cent. In the given example, however, under the suppositions made, the difference between actual and calculated maximum moments is less than 15 per cent. What it actually is, can be determined by the use of a similar corrected theory, but would involve great labor, without bringing us much nearer to the actual strains. Indeed the chords being not theoretically varied, the discrepancy is small as compared with other shortcomings of the usual theory.

This tedious correction was made in some instances, as for the Vistula bridge, near Dirschau, and the error was taken into account in the design of the Krementschug bridge over the Dnieper in Russia. This bridge was designed in 1866, with all possible economy.* It consists of two parallel and separate structures, one for roadway and one for double railway tracks. There are four

* By H. Sternberg, Civil Engineer and Professor in Karlsruhe. Through his kindness the author received copies of the design, strain sheet and estimate. so that the figures quoted deserve the more confidence, as referring to the completed work of not a mere theorist. but one of much practical experience in matters of design and manufacture. The bridge was, however, built upon another design.

through spans, 118 metres or 387 feet between centres, bridged by two pairs of continuous girders. The calculation was based on a dead load of 3,710 and on a live load of 3,600 kilos per meter for each track (respectively 2,480 and 2,400 pounds per foot). The weight of the iron work proper, amounts to 2,340 pounds per lineal foot of each track. There are two trusses, 10.5 meters or 34.5 feet deep, which is a depth $\frac{10}{113}$ of the span. The webs are designed as stiffened lattice work arranged in ten systems, the diagonals running at angles of 45°. The diagonals of the meshes are 2 meters or 6.56 feet long, equal to the distance of cross bearers of 4 feet 2 inches depth. The rail-stringers are of wood.

The following weights have been calculated:

Chords of one span of two tracks............	946,000	pounds.
Webs " " " " "	508,000	"
Bracing and floor beams, &c., ...	360,000	"
	1,814,000	

Or 2,343 pounds per foot of each track. These weights are obtained under maximum strains of 8,500 pounds to the square inch (600 kilos per square centimeter) both for compression and for tention of compressional

liagonals and chords as well as of parts under ;ension, for which latter ones the net areas ıre considered.

The lightness of webs was much furthered ɔy the diagonal system and by adopting the ;ame strain throughout as far as practicable. Րhis lightness of course is secured, but only ınder sacrifice of certainty as to the diagonal ȝtrains and of the postulate of scientific determination of these strains. Nevertheless, the Krementschug design is certainly one of the best proportioned and most economical *lattice* bridges, and gives evidence of its being designed by an engineer who knows how to appreciate expense of manufacture in the mill as well as in the shops and field. The bridge when compared with other European structures is light, and comparatively much ligther than the Mainz bridge on Pauli's plan, so much the more since the latter is designed for 20 per cent. lighter live load and for 33 per cent. greater strains. Yet compared with single quadrangular trusses, designed with the most improved American details, the Krementschug bridge does not show economy in material, and in regard to manufacture and erection it cannot compete with single spans.

A quadrangular double track truss bridge of proper proportions and details, 400 feet span, can be built with the *same* weight of iron per lineal foot for the same live load, and the average maximum strain.

In the calculation of the Krementschug bridge, the moments and shearing forces were first determined according to the usual theory, whereupon the correction was introduced due to the varied moment of inertia of the structure. The maximum moment over the central pier under full load of the bridge amounted to 12,300,000 kilogrammeters, but was corrected to 14, 420,000 kilogrammeters. The maximum moment, however, towards the middle of each span was only reduced by $2^1/_2$ per cent. The chords, therefore, by the correcrection were increased, as also were the web strains, slightly*

We shall now apply the formulæ gained

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* The design of Herr Sternberg's bridge has this advantage over many newly built lattice bridges, that the lengths of compressional members are so chosen as to permit their being proportioned for crushing and not for crippling. The chords, 40 inches wide and 32 inches deep, are made only 6 feet long between panel joints, are braced laterally every 6 feet in the bottom chords (cross-bearers), and every 12 feet on top, so that the top and bottom lines of each chord are held in position. The compressional diagonals form dia-

for the calculation of the influence of the webs on the reactions p of our two continuous spans of 200 feet. We shall only consider one part of this labor by supposing the bridge to be fully loaded while an exact calculation would require to suppose all those modes of loading which we have examined under consideration of the chords, according to the common theory.

The chords of our design are almost of equal section. For the sake of simplicity we therefore apply the general formula for the angles γ and δ under supposition of an average chord section. The dead load is 1200 pounds and the live load is 2240 pounds per foot. There results for deflection:

$$\gamma = \delta = \frac{3440 \times 200 \times 200 \times 200 \times 2}{2 \times 24 \times 30,000,000 \times 25 \times 25 \times 25} = \frac{5.87}{2400}$$

phragms between the vertical chord plates, 32 inches deep. for panels of 13 and 15 feet, the chords must be proportioned against crippling, their radii of gyration must be correctly calculated and inserted in the formula. Thin vertical plates of channel shaped chords should be secured in position by diaphragms, and the lateral bracing should be properly calculated and proportioned. The radius of gyration for channel shaped chords of lattice bridges is very small, and the chord sections will increase considerably, if the specification is duly enforced.

where the divisor 2400 is the length of the span in inches. This angle of *deflection* $\frac{5.87}{2400}$ must be increased by the influence of the posts and diagonals. The result of the calculation is $\frac{3.79}{2400}$ showing a correction amounting to about 61 per cent. of the angle due to the chords alone. The angle of *elevation* caused by the chords under action of the total force $p = 42,630$ pounds, such as found with the ordinary theory, equals

$$\frac{42.630}{3} \cdot \frac{200^2}{30,000,000.\ \frac{25^3}{2}} = \frac{5.82}{2400}$$

This value of elevation is so nearly equal to $\frac{5.87}{2400}$, found to be the angle of deflection due to the chords alone as caused by the full load on the two spans, that—were it not for the web system—we should feel very much satisfied with the exactness of the theory.

But the force $p = 42,630$, also causes extensions and compressions of the web members which result in the angle of elevation

$\frac{1.7}{2400}$ amounting to 30 per cent. of the angle caused by the chords. We have now these angles:

	Caused by chord.	Caused by web.	Total.
of deflection	$\frac{5.87}{2400}$	$\frac{3.79}{2400}$	$\frac{9.66}{2400}$
of elevation	$\frac{5.82}{2400}$	$\frac{1.70}{2400}$	$\frac{7.52}{2400}$

The total angles should be equal, but they differ by 29 per cent. The angle of elevation is too small, in other words the force 42,630 is by 29 *per cent. too small*, or hence the maximum moment over the middle pier which we found to be 8,130,000 pound feet is by 29 per cent. too small, and the chord sections at this point should be 49 inches instead of 38 inches. The webs are too strong at the end piers and too weak at the central pier. The chords in the middle of the spans such as designed are too large. The point of contrary flexure is nearer to the center of each span than anticipated. The force p being increased from 42,630 to 55,000 pounds., the end reaction A under full load from 115,036 pounds decreases to 102,666 pouuds.

While according to the common theory

0.375 of the total load should be carried by the end piers, the corrected theory only gives 0.34; in other words, instead of $^3/_8$th only $^3/_9$th of that load are carried by the end piers and the maximum central moment for two equal continuous spans under the ordinary theory expected to be $\frac{1}{8}\, p\, l^2$ becomes only $\frac{1}{6.2}\, p . l^2$.

We should now also calculate the influence of the web under other suppositions as to the position of live load. We then should have to calculate anew the strain sheet, we should have to correct the sections, and finally make the whole calculation over again, when again the claims for economy of continuous bridges were to be examined.

We will not enter into this labor, the inevitable conclusion being that the common theory is not sufficient for the calculation of continuous skeleton structures. Its use is confined to the proportions of homogeneous shallow plate girders. In some instances continuous rolled beams and plate girders of uniform section may be used with advantage in buildings and for floors of bridges if vertical stiffness must be secured and if the head-room is very

limited. But true economy always points to single spans.

The most economical structures require but very little calculation, so that estimates can be made within a few hours without formulæ or drawings. A practical engineer will get along without any formulæ. All that is necessary towards making a good estimate is a piece of paper and a pencil in the hand of a bridge engineer, who in the school of practice has learned to sift rubbish, both analytical and graphical, from the few principles of natural philosophy which are really needed, which are commerciably applicable and from which, by plain reasoning, special rules readily can be derived whenever desirable.

It must not be understood as if analysis were considered to be worthless; on the contrary analysis, *if not superficially applied*, is a most powerful *thought aiding machinery* and always logically gives the correct answer to a question, but it does not criticise the hypotheses, unless several contradictory hypotheses had been underlaid to the calculation. The proper appreciation and limitation of the power of analysis from an engineering point

of view most lucidly has been given by the late Professor Rankine in the preface to his applied mechanics, which we recommend to the readers.

VI.—Esimate of possible Irregularities in the Strains of the two continuous 200 feet Railroad Spans investigated in the previous paragraphs.

We will now consider the irregularities caused in the strains of continuous girders if, from any reason, they do not fit to their bed plates. For this purpose, we refer to Fig. 11 and to Eq's (*VIII*), and assume *first*, that the masonry of the middle pier has settled one inch. We have for the moment of correction and for the correction of the pier reactions $M = \frac{3\,E\,I\,d}{l^2}$ and $p = \frac{M}{l}$; E being the modulus, assumed at 30,000,000 pounds, I the average moment of inertia, equal to say 7,200 inches pounds, and l the length of the span in feet, consequently, $M = \frac{3.30\,000\,000.7\,200.1}{12.200.200} = 1{,}350{,}000$ pounds feet.

This moment of correction will produce pressure in the top chords and tension in the bottom chords over the middle pier. The *re-ac-*

tion of each end pier will be increased by $\frac{1,350,000}{200} = 6.750$ pounds. The bridge being fully loaded, by reason of the settled pier the moment over this pier will *decrease* from 8,430,000 to 7,080,000, that is, by 16 per cent.

If the bridge were fully loaded only on one span, the maximum moment within this span would *increase* by $p \frac{5}{12} l = 6,750 \,.\, \frac{5}{12} \,.\, 200 = 562,000$ pounds feet. The maximum moment of the fifth panel (see Plate) was 5,952,000 pounds feet, and increases to 6,514-000 pounds feet. This is an *increase* of $9^1/_2$ per cent., or about as much as it was expected to save in the chords under application of the theory of continuity.

If from any reason—defective construction in the shops, or the middle pier being built too high, or the end piers having settled—the bed plate on the middle pier should lie comparatively too high by *one* inch, the central maximum moment would be increased by 1,350,000 pounds feet, and the total strains over the middle pier would be increased by 44,000 pounds, or by 16 per cent.

of their calculated values, and the moments within the span would increase or decrease correspondingly. For every other inch of difference between bed plate and girder bearing the same correctional strains would arise, for Eq's (*VIII*) teach that the corrections are proportional to the values of elevation or depression.

It follows then, conclusively, that the introduction of continuous girders requires the best class of foundation and masonry for the piers. Alone from this reason, practical engineers would not like to use delicate superstructures like continuous girders, even if these would afford some economy of material, which, as we have seen, is not the case.

It was proposed, long ago, to improve continuous girders by weighing the reactions. But the question arises, whether this improvement, which involves some additional cost, is any longer necessary when we know that the theory is of so little practical value. It is also questionable whether a continuous girder, regulated by scales for one mode of loading, would still be properly adjusted under any other position of a moving train; for we know

that with each other position of the load, other diagonals will come into action.*

We will next examine the influence of the sun on continuous bridges whose upper or lower chords are covered by roadway planks or otherwise.** In this climate, the power of the sun is great, as any one may feel on a hot summer afternoon by laying his hand on iron exposed to the direct rays of the sun. The difference in heat of iron thus exposed or shaded may be 30° or 40° Fahr. Suppose, therefore, the bottom chords of our 200 feet spans to be covered by planks, what would be the correction needed for a difference of temperature equal to 30° Fahr.?

Under this supposition, the girders, considered without weight, would rise so as to form part of a circle whose radius is $150{,}000 \cdot \frac{25}{30} = 125{,}000$ feet. The rise in the centre of a

* For an ingenious mode of regulating the reactions of the Boyne bridge, see Theory of Strains by B. B. Stooey. Vol. II., p. 460.

** Single span bridges without counter diagonals, whether with one or more web systems, are entirely free from this influence. Those with counters experience in their region extra strains of abont 2,000 lbs. per square inch, immaterial since the counters and diagonals in the center of single spao bridges are made stronger than indicated by calculation.

chord of 400 feet woud be, consequently, $\frac{200 \;.\; 200}{250.000} = 0.16$ feet, or 1.92 inches, which, from Eq's (*VIII*), is known to cause a moment $M = 1{,}350{,}000 \;.\; 1.92 = 2{,}592{,}000$ pounds feet. This has a tendèncy to reduce the momént over the middle pier, which, for the unloaded bridge, was found equal to 374,000 pounds feet, leaving still a pressure in the bottom chords over the middle pier of 393 pounds per square inch of section. The moment M, 2,592,000 pounds feet, would cause additional pressure on the end piers of 12,960 pounds, and additional strains in end diagonals equal to about 10 per cent. of their maximum values.

If one span were fully loaded the maximum moment of 5,952,000 would be increased by $13{,}000.\frac{5}{12}.\,200 = 1{,}083{,}000$ pounds feet, which is 18 per cent.

At the points where the moments change from positive to negative comparatively very great moments would be produced. Thus, at the third panel-joint from the middle pier, the greatest positive moment is 2,200,000 pounds feet, which would be increased by

$13{,}000 . \frac{3}{4} . 200 = 1{,}950{,}000$ pounds feet, so that the greatest positive moment at that point would be 4,150,000 pounds feet. In case the top chord were covered by floor planks, the moment M, 2,592,000 pounds feet, would cause tention in the top and compression in the bottom chords over the middle pier. The maximum moment, 8,430,000 pounds feet, would be increased more than 30 per cent.; so that each degree Fahr. would cause one per cent. of additional strain. The negative moment at the second panel-joint from the middle pier would be increased from 3,230,000 to 5,400,000 pounds feet, that is, by 67 per cent.

If the temperature in the top chords were raised to 40° Fahr. the central bearing could no longer act under the dead load only, for the truss would be ⅝ inch above the bed-plate.

Arch bridges without hinges must be designed under the theory of continuity, with its defects and unfounded suppositions. Some of the objections against this theory as here applied have peculiar force—as the difficulty of proper manufacture, of close fit to the piers, and especially the influence of tem-

perature. A few exclusively theoretical writers—on the authority of Oudry's experiments, and of thermometric measurements at the Tarascon bridge in France—deny the influence of heat on iron arch bridges with flat bearings. But they ignore the experience gained with the Theis bridge in Hungary, which might set at rest experiments with the thermometer.* This bridge changes its bearings on the abutments daily, and it is observed that the pressure moves from the lower chord bearing to the upper and back again. The bridge being by design very stiff, leaves its thrust bearings in winter, and unloaded, acts as a beam. It was anchored

*Mr Stoney thus remarks about the effect of temperature: The rise in the crown of one of the cast iron arches of Southwark bridge for a change of temperature of 50° Fahr. was observed by Mr. Rennie to be about 1.25 inches; the length of the chord of the estrados is 246 feet, and its versed sine, 23 feet 1 inch, and accordingly the length of the arch, which is segmental, is 3.020.8 inches. The range of temperature to which open work bridges, through which the air has free access, are subject in this country, seldom exceeds 81° Fahr. The range of temperature of cellular flanges, may, however, exceed that mentioned above, as Mr. Clark mentions that the temperature of the Britannia tubular bridge, before it was roofed over, differed "widely from that of the atmosphere in the interior, for the top during hot sunshine has been observed to reach 120° Fahr., and even considerably more; and on the other hand, a thermometer on the surface of the snow on the tube has registered as low as 16° Fahr."

to the abutments the next summer after this observation was made, but during the following winter the piers commenced to move, wherepon the connections were removed. This example illustrates one of the practical difficulties inherent to continuous girders.

Most all European continuous bridges, as well as single span bridges, have either their bottom or their top chords protected from the sun's rays. The high iron viaducts in Switzerland and France (Freiburg, Busseau, Cére, &c.,) have superstructures of 7, 6 and 5 continuous spans, of which the top chords are protected by flooring. But we have seen that the strains of continuous bridges, whose bottom chords are overheated, will be greatly disturbed, and that for a difference in temperature of 30° Fahr. addition-strains of 30 and even more than 50 per cent. may arise. We are not aware that this has been considered in the construction of continuous bridges.

Stone arches are affected in the same way as iron arches. With increased temperature the crown rises, and joints in the parapets over the crown open, while others over the springing close up. The reverse takes place in cold weather. In addition to the longitudinal movements to which all girders are subject from change of temperature. tubular girders move vertically or laterally whenever the top or one side be-

comes hotter than the rest of the tube. Referring to the Britannia tubular bridge Mr. Clark states that "even in the dullest and most rainy weather, when the sun is totally invisible the tube rises slightly, showing that heat as well as light is radiated through the clouds. In very hot sunny days the laterial motion has been as much as 3 inches, and the rise and fall 2.3 inches. These vertical and lateral motions have not been much observed in lattice or open girder work, no doubt because the air and sunshine have free access to all parts (?) and thus produce an equable temperature."

James Hodges, in his work on the Victoria bridge, remarks: "In building the tubes the greatest increase of camber which occurred in one day consequent upon the difference of temperature between tops and bottoms of tubes was 1¼ inch; the barometer on the top reading 124°, in shade at bottom 90°, making a difference of 34°. The thermometer during the previous night was as low as 57°. It is therefore only fair to infer that as the bottom was in shade it would not be of the same temperature as the atmosphere, and that the increase of camber of 1¼ inch, was due to difference of temperature of probably as much as 50° Fahr.

The greatest longitudinal movement of roller beds was, for 258 feet 3¾ inches, due to a variation of from — 27° to + 128° = 155° Fahr. The greatest lateral movement caused by temperature was 1¼ inch.

The Victoria bridge is roofed over with timber and tin, and the temperatures measured probably were only those of the atmosphere, but not those of the iron. The girders being only imperfectly continuous, calculation of deflection of these girders is still less reliable than of theoretically designed work. Also draw-bridges (continuous over two spans) move sideways in consequence of one truss being more heated than the other. The side movement of a 360 feet draw was noticed as much as 1¼ inch and caused some trouble in locking. About equal vertical movements were noticed of a 360 feet draw. with planked floor. Mr. C. Shaler Smith (Transactions, vol. III, page 131, &c.) noticed alterations of the height of support caused by the sun and reversedly by cold winds—of as much as 1 inch. Mr. John

Griffen informes us, that a 134 feet draw on the Philadelphia, Wilmington & Baltimore R. R. was so much affected by the unequal heating that it could not be turned, the deflection being about ⅝ inches. This he remedied by covering the top chord with wood.

We still have to compare the deflections of single and continuous spans. According to the theory, the deflection of a single span reaches its maximum under full load, which then is $\frac{5}{384}\frac{Pl^3}{EI}$ in which P is the total load. For two continuous spans the maximum deflection occurs if only one span is loaded, and equals $\frac{4}{384}\frac{Pl^3}{EI}$. These applied to the two hundred feet spans give a deflection under live load for the continuous span of 2.08 and for the single span of 2.05 inches. The deflections are equal, so that also from this consideration there is no reason to prefer continuous girders. It would not be desirable to adopt these girders on this score even if the deflection were one-half less than for single spans, for we build single spans with a camber, so as to make the floor just level under full proof load.

It is only with rafters and purlins of roofs made of shape iron of small depth that the

consideration of continuity may lead to constructive advantages, and in some instances this consideration may be valuable in the construction of floors of bridges, but for bridge trusses of which we can vary the sections and can adopt a suitable depth, the question of reduction of deflection never arises, and continuity on this score need not be resorted to, either with girder or with arch bridges.

Finally, we find that the large continuous bridges over the Vistula of six spans, over the Rhine at Cologne, over the Dnieper at Krementschug, over the Danube at Pest, each of four spans, from 321 to 418 feet long, are only continuous over one support. *Theoretically*, there is an advantage as to the average moment of flexure of girders stretched over more than two spans, but also a greater quantity of total webs needed, and practically there is a loss of material at each change from concave into convex flexure. For two spans, there are two such regions, for three spans their are four, and for four spans there are six. Therefore, practically two spans are about as favorable in regard to the moment as three or four. Any how, the longitudinal expansion of the girders limits their length. For a maximum

change of temperature equal to 150°, the change of length amounts to one-thousandth of the span, which for four 400 feet spans amounts to 1.6 feet, so that at one end of such a bridge, provision must be made for a movement of 0.8 feet or 9.6 inches.

This consideration probably has limited the application of continuity to only two spans of great dimensions. There are, however, some bridges in Germany of seven and even nine (small) continuous spans, and in France and Switzerland are some large viaducts of five, six and seven spans of 150 feet average length. Most all of these (viaduct of Freyburg, Busseau, Cère, a bridge in Vienna, &c.) were built by iron works in France, and by Benkisser's method, the girders have been rolled over the piers, a method probably preferred on account of facility of erection, the works having a full plant for the purpose. On the other hand, most builders with whom the number of their orders is a consideration only second to quality and reputation, erect continuous girders on carefully built false works.

Before closing this examination it is still mentioned that the correctness of the theory of continuous girders, or of the execution of

the work, the moduli previously being experimented upon, can be examined by comparing the actual deflection with the calculated ones.

Namely, in case the reactions being correctly calculated, also the strains must be correct, and hence the extensions and compressions, and consequently also the deflections must agree with the calculated ones.

The French engineers, who developed and applied the ordinary theory, knew this very well, and as cautious engineers and thorough theorists, did not accept the theory before they compared the deflections. They found them to agree with what they considered the requisite closeness of approximation. But those structures were shallow plate girders built on carefully prepared false works, and hence the suppositions of the theory were much more nearly fulfilled than happens with deep skeleton trusses.

The deflections, as it were, would therefore be the test stone of the correctness of the calculation.

One of the continuous structures of the Swiss North Eastern R. R. (which is considerably heavier than we build equally strong and equally long single spans,) is the *Ergolz*

Bridge near Augst, consisting of four spans of 100, 122, 122 and 100 foot, 11' 2'' deep.

The test-load consisted of locomotives 2,667 pounds per foot. The theoretical maxima deflections (the moduli we learn to have been experimented upon, but to what extent is not known) were :

14, 19, 19 and 14 millimeters $\left(\overset{\text{mm}}{1} = {}^{1}/_{25}{}'' \right)$

The actual maxima deflections under strains of mean velocity were :

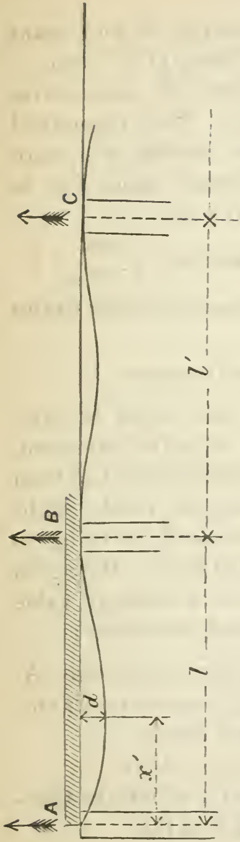
11, 10, 11, 12 millimeters.

The calculation hence was out of the way by as much as 90 per cent. to, in the minimum, 17 per cent. If *all* actual deflections had been less in the *same proportion*, the result might have been *attributed to the modulus* being so much larger than calculated upon. We again assume the modulus to be a constant, value in the following algebraical deduction :

Let A B C represent a continuous bridge, A. and B, &c. representing reactions under uniform load on one or more spans.

l denotes the length of span A. B.

x_1 denotes an abscissis for which the theoretical deflection is d, and this



x_1 denotes an abscissis for which the actual deflection is d_1.

A denotes the theoretical reaction at A.

A_1 Denotes the actual reaction at A.

p in the load per foot on A B being fully loaded.

E is the assumed constant modulus of elasticity.

I is the constant moment of inertia of the bridge.

We have the well known equation :

$$E\,I\frac{d\,y}{d\,x} = A.\frac{x^2}{2} - p.\frac{x^3}{6} +$$

(Constant $=$ C)

and

$$\text{E I } y = \frac{\text{A}}{6}.\ (x^3 - l^2x) - \frac{p}{24}(x^4 - l^3x)$$

oecause :

for $x = l$, $y = o$, and $\text{C} = -\left(\text{A}.\frac{l^2}{6} - \frac{p}{24}.l^3\right)$

We now can calculate the deflection d for a point whose abscissis$= x_1$.

Theoretically we ought to get :

$$\text{E.I.}d = \frac{\text{A}}{6}(x_1{}^3 - l.^2x_1) - \frac{p}{24}(x_1{}^4 - l^3x_1)$$

but practically we do receive :

$$\text{E.I.}d_1 = \frac{\text{A}_1}{6}(x_1{}^3 - l^2x_1) - \frac{p}{24}x_1{}^4 -{}_1)$$

Now, E, I, x_1, l and p *being constant* quantities, A_1 cannot be equal to A, because d_1 is not equal to d. They only could become equal if E were different from what it has been supposed in the calculation. This supposition here falls away, because the actual deflections of the Augst Bridge do *not* correspond *proportionally* with the theoretical ones. Hence, by subtracting the equations, we get :

$$\text{E I } (d - d_1) = \frac{\text{A} - \text{A}_1}{6}(x_1{}^3 - l^2x_1)$$

in words :

The difference of actual and theoretical deflection would be proportional to the difference

of actual and theoretical reaction, provided we were right in using the theory in the calculation of continuous trusses. And we find :

$$A_1 = A + \frac{d_1 - d}{(x_1^3 - l^2 x_1)} \,.\, 6 \,.\, E \,.\, I.$$

The actual reaction is greater than the theoretical reaction by

$$\frac{d_1 - d}{x_1{}^3 - l^2 x_1} \,.\, 6 \,.\, E\, I,$$ or consequently.

The actual reaction not being equal to the theoretical one, the actual strains are not the theoretical ones, or the calculation of theory does not correspond with reality.

Whether this proves incorrectness of the theory, or improper execution, is not open to a conclusion from the mere experimental results without the values E and I. But it is sufficient to know that remarkable differences of theory and practice do exist, and may be expected again.

VII.—Recapitulation and Conclusions.

1°.—The mere theoretical calculation of the curves of moments and shearing forces of girders or arches without proper consideration of proportions, details and cost of manfacture, is exceedingly fallacious, and this fallacy will be the greater, if the theory by

which the moments and shearing forces are calculated, stands upon false premises.

2°--There is *theoretically* no saving in continuous bridges over most economically arranged single spans. Whenever single spans can not be built economically there is the place for continuous bridges of which the points of reversion of curvatures are fixed by hinges.

3°.—The theory of continuity is based on the hypothesis of a constant modulus of elasticity, which, as proved, does not agree with the nature of the material. It has been shown that the modulus of wrought iron varies from 17,000,000 to over 40,000,000 pounds per square inch.*

4°—Even if it were assumed that the modulus had a constant value, still a correct theory would require that there be but one system of diagonals in the web of a continuous girder. Under an arbitrary supposition, the strains in the diagonals and posts of continuous girders with two or more systems cannot be calculated, but only guessed.

5°.—The theory neglects the influence on the moments and shearing forces caused by

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*Page 161.

the deflections due to the extensions of the web ties, and to the compressions of the web struts.† The theory also needs a correction if the chords are varied.

6°.—The correct application of the principle of continuity involves an exceedingly tedious labor, and, if generally introduced, would greatly impede the business of bridge construction in this country.

7°.—In the determination of the section of chords and webs, it must be considered that a member exposed to tension as well as to pressure, must be proportioned to resist the maximum tension plus the maximum pressure.

8°.—Continuous girders require very accurate workmanship, both in the shops and in the field, which, if exacted by the inspecting engineer, will cause a greater expense than that for single spans. The connections at the points where the strains change from the positive to the negative must be made with more care than if tension or only compression had to be resisted. Especially, in case of riveting, the holes must in the field be

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† We have proved that this influence is considerable, and upsets the common theory.

rimmed to match perfectly, the rivets placed more closely and driven thoroughly.

9°.—The foundations and masonry of piers on which continuous girders shall be placed, must be of excellent quality. Single span trusses may, without injury, be placed on piers which have settled several inches; but this is not the case with continuous girders. Engineers contemplating the use of continuous girdere should realize the necessity of this provision, and previously estimate the additional cost of substructure.

10°.—If it is intended to roll continuous girders over the piers, ordering and inspecting engineers should examine carefully whether the contractor has calculated the extra strains arising from the weight of the projecting cantilever, has properly reinforced the posts and introduced additional diagonals and chord material at the points of change of flexure.

11°.—Continuous girders improperly built or placed on their bed plates, have to resist greater strains than contemplated, which, for one inch difference in height of location of bed plate, on the middle pier of a 200 feet span, is increased by 16 per cent.

12°.—If the upper or the lower chords of a continuous bridge are protected from the direct heat of the sun, the strains are much disturbed and (for 30° difference of temperature) may, over the middle pier be increased 30 per cent. and at the points of change of flexure more than 50 per cent., and the structure may even rise from the middle piers, notwithstanding its dead load.

13°.—The proportions of depth of span to height depend essentially on the system and on the details used. The lighter theoretically and practically the web can be made, the greater the height can be chosen, which is only limited by the practicable length of web members and by the calculation of the strains, sections and weights due to the effect of wind. Practically, the best depth is obtained if an additional foot increases the weight of the total structure. Continuous girders, requiring more material in their webs than single spans, cannot be built as high. European bridges having been built too shallow for single spans, as far as economy is concerned, were better proportioned when built continuously. This is one of the reasons why, in

Europe, continuous bridges proved to be the lighter.

Properly proportioned single spans on the same system, at least should be no heavier.

14°.—We have found by investigating the example of two 200 feet spans that properly designed single spans with American details, are actually lighter than continuous girders. The bridge of Buda-Pest and the Krementschug bridge are examples of large continuous structures of economical European construction, but they do not compare either in cheapness or quality with single spans, well proportioned, having the most scientific American details.

15°.—Continuous bridges deflect as much as single spans of correspondingly greater depths. It has now been proved that the theory of continuity, most interesting as it is in a scientific point of view, nevertheless forms only a part of pseudo-science; being based on false suppositions. it is too delicate in execution and under use, and finally, because it is not economical. Practical constructions are designed with a certain factor of safety. The truer the theory, the easier its suppositions can be fulfilled, the less its re-

sults are modified by disturbing influences; the less it is influenced by the unreliable incidents of the application of human labor: the more reliable a construction will be, and the smaller the factor of safety may be taken.

It has been shown, that by theory we cannot gain greater perfection in practice, unless we constantly are comparing the results of our deductive investigations with experimental facts.* Results of such experiments on executed continuous girders are not known. All that we have are some notes on deflections of finished bridges under test loads. We usually learn that the deflections were much less than expected or calculated, which is communicated as a proof of the excellency of workmanship, as if workmanship could reduce the extensions or compressions of iron —in other words, could raise the modulus. In this country, continuous girders have only been used for draw-bridges. The calculations of the strains of these continuous girders are still more complicated, more delusive, and more untrustworthy than those made for fixed bridges, principally on account of the

* Compare what Mr. B. Baker says, pages 221, 228 and 313, "Strength of Beams, Columns and Arches. London. 1870.

compressibility of the turning apparatus and masonry, as well as on account of irregularities of end supports.

It is not intended to enter into these mathematics. It only is mentioned that, probably, Prof. Sternberg was the first engineer who applied the theory of continuity to the various suppositions as regards dead load of fixed and swinging draw, of partial and full live load, the draw being screwed up at ends or loose, &c. The pivot bridge, by him was considered as a continuous girder over three openings, two large outer spans and a short middle part. Herr Sternberg applied the formula thus obtained to the draw of Kustrin in Prussia.* The central part of the draw really constituting a separate part of a lattice beam composed of chords and lattice diagonals, this calculation was justified. With large draws,

* His whole investigation was published in the report of the Kreutz Kustrin R. R. of 1857. In his lectures the professor gave it in all its essential features, when he also mentioned the idea of weighing the reactions of continuous girders. In this country Messrs. Channte and Morrison in their work on the Kansas bridge applied the theory to continuous draw bridges. Mr. Shaler Smith in a very lucid article in the transactions of the Am. Society Co. E., 1874, has explained his mode of building continuous draws without end reactions when unloaded, as first introduced by him.

as built in this country, the calculation based on three openings can be reduced to that for two spans so long as the two centre diagonals, only necessary to give stiffness during the movement of the swing bridge, are provided with elastic sling loops, or any other suitable elastic medium. The half loaded draw will depress somewhat the drum, the wheels and even the masonry, and the light diagonals being incapable of taking up any great amount of strain without stretching, the other bearing on the round pier will remain in action. The two chords above the round pier will sensibly experience the same amount of stress. It is even admissible to slacken these diagonals when the draw is fixed, so that the bridge may act by a scientifically correct general arrangement of modified continuity, and before the bridge is to be turned, by some arrangement, the diagonals might be brought into action. By such construction the greater part of the weight would not be thrown on only a few wheels. The great and unnecessary complicity of the calculation of a bridge resting on four supports can be dispensed with, so much the more since the theory

of continuous trusses deserves but little confidence.

In the discussion on this subject (Transactiens American Society of Civil Engineers, 1876, Vol. V. pages 227 and 228), the author has proved, mathematically, the correctness of this construction. It had already been put under test in Mr. A. P. Bollers draw span of 258 feet over the Hudson at Troy, New York; which, having no center diagonals at all, even swings around without central bracing, simply relying upon the stiffness of the riveted chord. However, it sways a little more than the designer wished. Continuous draws can be entirely avoided by building draws consisting of two single spans to be united when the water way has to be opened. This construction was studied and worked out in details at the same time by the Keystone Bridge Company, and by the author. The detail constructions differ in that point, that the Keystone Company applies hydraulic presses at the ends of the single spans to be worked from the centre, so that the ends being raised the tensile central top chord bars, with oblong pinholes, are released of their tension, whereas, the author raises the ends

from the centre pier by shortening the distance between the central top pins, or by lengthening the central bottom chord by means of inserted hydraulic presses. (For sketches and details see the discussion referred to above).

The investigation which is now finished will surely not impair confidence in the construction of bridges whose design is exclusively based on the plain unmistakable law of the lever, which can be calculated in a short time, be easily manufactured and erected.

If engineers wish to build continuous girders, they will do better to use continuous bridges with hinges, (one kind of such structures was first proposed by Professor Ritter, hinges in alternate spans were first patented by de Bergne in England in 1865, then reinvented by Gerber in Munich 1866, and by the author in 1867 in this country, each of these inventions having been made independently from the other,) when they will escape all uncertainties caused by defects of theory, by inequality of moduli, by several systems of diagonals, by inequality of heights of supports, of additional strains caused by heat of

sun, &c.* Practical men will welcome such simplicity, notwithstanding it may not satisfy a few mathematicians, because the problems connected therewith, to them, may not seem sufficiently interesting.

* Mr. C. Shaler Smith is just about finishing a bridge of this kind over the Kentucky River, consisting of 3 spans of 375 each. This system was chosen on account of facility of erection, but not for the sake of ecenomy, the weights proving to be the same as for single spans.

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